VORTEX IDENTIFICATION IN ROTATING TURBULENT RAYLEIGH–BÉNARD CONVECTION OF WATER

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Abstract We perform Direct Numerical Simulations of rotating turbulent Rayleigh–Bénard convection in water, assuming that the material properties are either constant (Oberbeck–Boussinesq conditions) or vary with temperature (non-Oberbeck–Boussinesq conditions). One of the most remarkable features of rotating Rayleigh–Bénard convection in is the generation of columnar vortex structures and their size is known to depend on the viscosity and diffusivity. We use various criteria to identify these vortices and analyse, in which way their extracted number, size and shape depends on the temperature-dependence of the material properties, the Rossby number and also the used identification criterion.

INTRODUCTION

The large structures occurring in the fluid flow on the Sun's surface, in the atmosphere and oceans of planets, including our Earth, are primarily driven by convection. The actual shape but also the efficiency of the heat transport are, however, significantly influenced by the Coriolis force due to rotation. Crystal growth and the ventilation of buildings and aircrafts originate within the same physical framework. Understanding these fundamental processes is thus not only utterly important for geo- and astrophysics, but also in industry. In an idealised way, all these systems can be represented as a fluid heated from below and cooled from above, the so-called Rayleigh–Bénard convection.

NUMERICAL METHODOLOGY

We perform Direct Numerical Simulations (DNS) making use of a 4$^{\text{th}}$ order finite volume code for cylindrical domains [7], solving the three-dimensional Navier-Stokes equations under Oberbeck-Boussinesq (OB) conditions. Additionally, we advanced the code by including temperature-dependent material properties [3], i.e. we are also able to study non-Oberbeck-Boussinesq (NOB) effects.
To investigate rotating turbulent Rayleigh-Bénard convection, we conduct simulations in a cylindrical cell with unity aspect ratio, filled with water (Prandtl number $\mathrm{Pr} = 4.38$) for Rayleigh numbers of $Ra \in \{10^7, 10^8, 10^9\}$. We study the influence of the Coriolis force by applying a weak background rotation, moderate rotation rates and fast rotation rates, i.e. Rossby numbers ranging from $Ro = 10.$ to $Ro = 0.05$. $Ro = \infty$ corresponds to the non-rotating case.

VORTEX IDENTIFICATION

It is commonly understood, that for $\mathrm{Pr} \gtrsim 1$ the heat transfer can be enhanced in a certain range of Ro due to columnar vortices, sucking fluid out of the thermal boundary layers, known as Ekman pumping [6]. However, the temperature-dependence of the viscosity and the diffusivity affect the size of the vortices and thus the efficiency of Ekman pumping, which in turn can lead to a further increase of the Nusselt number under NOB conditions [2]. We want to quantify the actual influence by further analysing the occurring vortical structures. Various criteria to identify vortices have been developed, most of them based on the velocity gradient tensor $\dot{A}_{ij}$. At any one point, it can be decomposed into a strain (symmetric) and a vorticity part (antisymmetric):

$$\dot{A}_{ij} = \nabla_j u_i = \tfrac{1}{2}\left(\nabla_j u_i + \nabla_i u_j\right) + \tfrac{1}{2}\left(\nabla_j u_i - \nabla_i u_j\right) = S_{ij} - \tfrac{1}{2}\epsilon_{ijk}\omega_k. \tag{1}$$

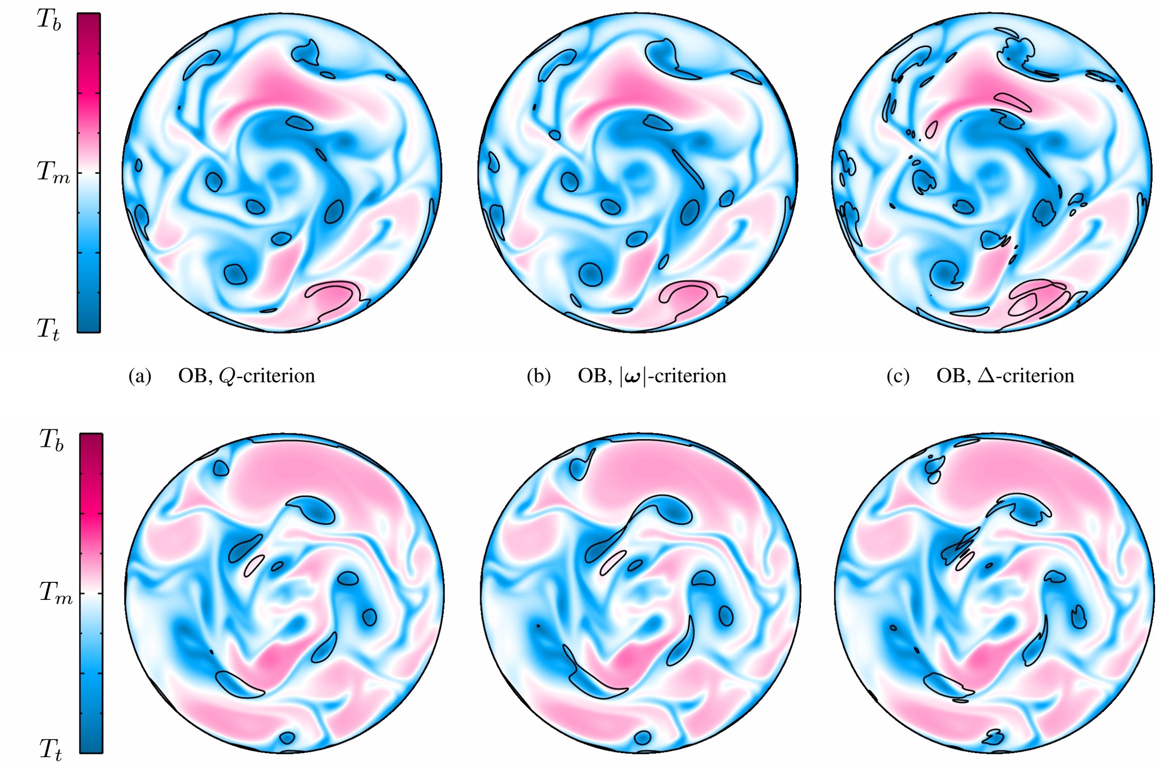
By solving its characteristic equation $\lambda^3 - P\lambda^2 + Q\lambda - R = 0$, with λ being an eigenvalue, the three invariants P, Q and R can be obtained. For an incompressible fluid they read

$$\begin{aligned} P &= tr(\dot{A}_{ij}) = \nabla_i \cdot u_i = 0, \\ Q &= -\tfrac{1}{2}\dot{A}_{ij}\dot{A}_{ji} = -\tfrac{1}{2}(S_{ij}S_{ij} - \tfrac{1}{2}\vec{\omega}^2), \\ R &= det(\dot{A}_{ij}) = \tfrac{1}{3}(S_{ij}S_{jk}S_{ki} + \tfrac{3}{4}\omega_i\omega_j S_{ij}). \end{aligned}$$

Hunt et al. [4] define a vortex as region where $Q > 0$ (Q-*criterion*), while Chong et al. [1] identify them by the requirement that $\Delta = (\frac{1}{3}Q)^3 + (\frac{1}{2}R)^2 > 0$ (Δ-*criterion*). Another commonly used criterion is based on a threshold of the magnitude of the vorticity [5] ($|\boldsymbol{\omega}|$-*criterion*).

PRELIMINARY RESULTS AND OUTLOOK

An example of the extraction of the vortices according to the different criteria for $Ro = 0.2$ and $Ra = 10^8$ is presented in figure 1. All criteria are essentially identifying the same vortical structures, however, their respective size differs. In the final contribution, we will show a statistical analysis on how the amount and the size of the vortices, as well as the Nusselt number, depend on the Ra and Ro, and in particular also how NOB effects influence them. We will also discuss the influence of the chosen vortex identification criterion.



(a) OB, Q-criterion (b) OB, $|\boldsymbol{\omega}|$-criterion (c) OB, Δ-criterion

(d) NOB, Q-criterion (e) NOB, $|\boldsymbol{\omega}|$-criterion (f) NOB, Δ-criterion

Figure 1. Cross section of the temperature field at the edge of the top thermal boundary layer under OB and NOB conditions for $\mathrm{Pr} = 4.38$, $Ra = 10^8$ and $Ro = 0.2$. The black contour lines indicate the circumference of the vortical regions according to the Q-, $|\boldsymbol{\omega}|$- and Δ-criterion.

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