NUMERICAL STUDY ON TURBULENT KINETIC ENERGY SPECTRA IN OSCILLATORY PIPE FLOWS

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Abstract We present kinetic energy spectra calculated from DNS data of oscillatory pipe flows at several combinations of the Womersley $Wo \in \{13, 26, 52\}$ and the Reynolds $Re \in \{3000, 12000, 48000\}$ number. For all Wo, the mean flow oscillation frequency contains most of the energy. Besides that, we find a $-5/3$ energy decay in the spectra, either close to the wall but not at the pipe centre for the conditionally turbulent case or at the pipe centre and close to the wall for the turbulent case. For the laminar cases, we observe small peaks in the spectra at higher modes, indicating instabilities already known from stability analysis.

INTRODUCTION

Shear flow turbulence in oscillatory pipe flows is of theoretical interest and practical relevance, since the onset of turbulence can drastically change the transport and mixing efficiency of these physiologically and industrially important flows. To supplement former theoretical and experimental investigations on the transition to turbulence in such flows, e.g summarised in [3, 1], we perform three-dimensional direct numerical simulations (DNS) of oscillatory pipe flows. For this, we solve the incompressible Navier-Stokes equations in discretised form by means of a fourth-order-accurate finite volume method described in detail elsewhere [2, 1].
The fluid is driven by a harmonically oscillating mean pressure gradient acting in axial direction. The resulting oscillatory pipe flow is fully characterised by the Reynolds and the Womersley number, defined as

$$Re = \frac{\hat{u} \cdot D}{\nu} \quad \text{and} \quad Wo = \frac{D}{2}\sqrt{\frac{\omega}{\nu}} \, , \tag{1}$$

where D is the pipe diameter, ν the kinematic viscosity of the fluid and $\omega = 2\pi/T$ the angular frequency of the oscillating bulk flow with the velocity amplitude $\hat{u}$ within one cycle T. In the present study, we focus on the evaluation of temporal and spatial energy spectra calculated from single point velocity time series and full instantaneous flow fields at different oscillation phases taken from DNS we performed for several combinations of $Wo \in \{13, 26, 52\}$ and $Re \in \{3000, 12000, 48000\}$. The numerical method as well as the used spatial/temporal resolution and the size of the computational domain was validated for a fully-developed statistically-steady pipe flow at $Re \approx 26000$, as described in [1].

RESULTS

For a choice of four out of eight performed DNS in total, fig. 1 presents velocity time series over $\Delta t = 80$ time units. Each time series starts after the respective flow has relaxed from its individual turbulent initial flow field and has more or less converged to a pure oscillation. On the right hand side, fig. 1 also shows the corresponding kinetic energy spectra $S_{u_x}(f)$ calculated from these time signals of the axial velocity component u_x by means of a fast Fourier transform. First and foremost, all spectra reveal, that the major part of the kinetic energy is contained in the frequency $f_0 = 1/T$ of the bulk flow oscillation for all Wo.
For $Wo = 13$ and $Re \approx 12000$, fig. 1a reveals fluctuations in the time series of u_x at the pipe centre line (blue), which are stronger and contain higher frequencies during phases of peak flow (PF), compared to phases of flow reversal (FR). That means the flow is more turbulent during PF due to higher bulk velocities and less turbulent during FR due to lower or even reversing bulk flows. The u_x history close to the wall (red) for the same case features distinct turbulent burst in most of the early deceleration phases (DC), while in the remainder of the oscillation cycle u_x stays more smooth and laminar. In both velocity trends, a phase lag can be observed with respect to the driving pressure gradient (black). This is an effect of the high $Wo = 13 \gg 1$, at which the core flow cannot follow the fast changing driving force due to inertia. Closer to the wall the phase lag is smaller due to stronger viscous effects.
In the corresponding energy spectra in fig. 1a we observe a $-5/3$ energy decay for a wide range of frequencies in the near–wall velocity spectrum (red), followed by a steeper decay for $f > 10$, as characteristic for the energy decay in turbulence. The energy spectrum of the centre line velocity declines with a steeper slope but with an interrupting plateau at $f_3 < f < f_5$. This indicates. that the core flow unlike the near–wall flow does not maintain a typically turbulent energy cascade.
On the other hand, for $Wo = 26$ and $Re \approx 48000$ the centre line spectrum as well as the near-wall spectrum in fig. 1d show a relatively good correspondence with a $-5/3$ energy decay in the higher frequencies, but the latter one with a marginally flatter slope. In this case, the core flow as well as the near wall flow is continuosly turbulent throughout the whole oscillation cycle. This can also be seen in the u_x history in fig. 1d, where, contrarily to fig. 1a, the velocity fluctuations increase during flow acceleration (AC) until PF and decrease again during DC for both positions.

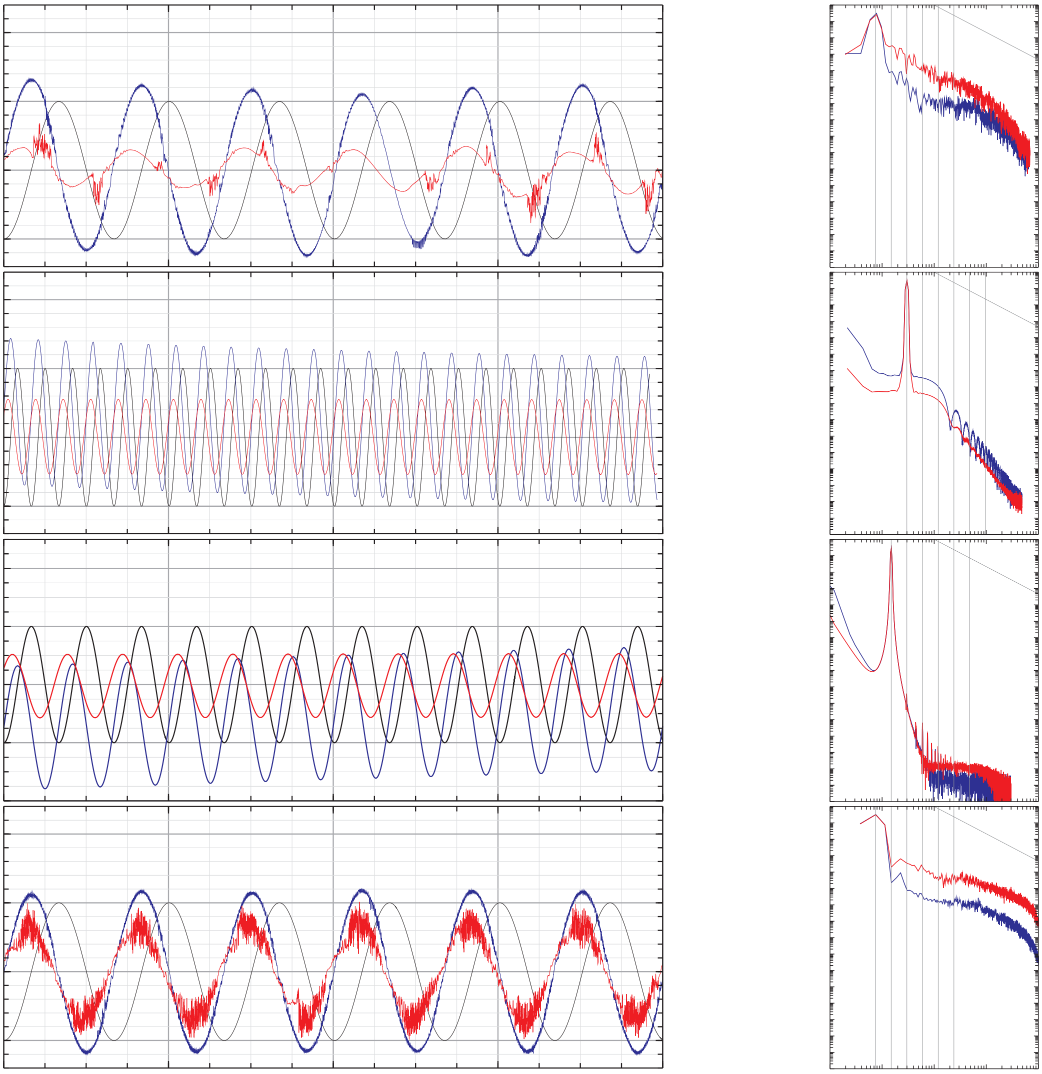

Figure 1. Single point time series of the normalised axial velocity component $u_x/\hat{u}$ (right) at the pipe centre line (red) and close to the wall (blue) and the corresponding kinetic energy spectra (left). The sets of dimensionless parameters are (a) $Wo = 13$ and $Re \approx 12000$ (b) $Wo = 26$ and $Re \approx 3000$ (c) $Wo = 52$ and $Re \approx 12000$ (d) $Wo = 26$ and $Re \approx 48000$, where the flow is driven by an oscillating pressure gradient (black). The vertical grey lines represent the bulk oscillation frequency f_0 and its multiples. The Kolmogorov $-5/3$ energy decay is represented by the inclined grey lines.

Fig. 1b and 1c show time series and the respective energy spectra for two cases, where the initially turbulent flow field completely laminarised, despite relatively high Re, due to the stabilising effects of high Wo. On the other hand, the energy spectra show distinct peaks at several multiples of f_0 (represented by vertical grey lines), which indicate flow instabilities already found by Trukenmüller [3] for similar combinations of Re and Wo.

At the conference we will additionally present spatial energy spectra calculated full instantaneous flow fields at several oscillation phases, to discuss and compare the energy decay for these different phases.

References

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