DNS OF BOUNDARY LAYER TRANSITION AT MACH 6

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Abstract 3D direct numerical simulation is employed for studying the flat-plate boundary layer instability evolving at high free-stream Mach number to study the non-linear interactions of the disturbances of different modes and the onset of the transition to turbulence.

INTRODUCTION

Laminar-turbulent transition in the boundary layer at high free-stream Mach number is greatly dependent on initial growth of small disturbances, at least if the level of fluctuations in the free stream is reasonably low. At subsonic and low supersonic speeds the boundary layer instability is governed by the well-known Tollmien–Schlichting waves. In supersonic boundary layer these growing disturbances are usually referred to as the *first mode instability.* First-mode disturbances are essentially vortical. In compressible case, the most unstable first mode disturbances are oblique: they propagate in the boundary layer at an angle in transverse direction *Z*. As the free-stream Mach number increases, their growth rates in downstream direction *X* dramatically decrease. Linear stability theory was applied by L.M. Mack [1] to study the boundary layer instability at high Mach numbers. He found a *second mode* of growing disturbances, acoustic in nature, which have higher growth rates compared to the first mode, and is essentially two-dimensional, i.e. the most unstable second mode disturbances propagate plainly downstream.

The problem with these second mode disturbances is that, according to linear stability theory with consideration of non-parallel effects, second mode disturbances do not live long: as the boundary layer thickness (measured along the normal coordinate *Y)* increases downstream along *X*, initially unstable high-growth-rate second mode disturbance of a given frequency quickly becomes stable and can no longer grow. We performed our own linear stability theory calculations using the formulation [2], and found exactly that. On the contrary, the first mode disturbance of a given frequency, though less initially unstable than the second mode, can still be unstable farther downstream. The question then arises, how the transition happens in reality. We can completely discard the first mode due to its low growth rates, and depict our instability scenario as the cascade of planar second mode disturbances involving lower and lower frequencies. We can as well use just one single-frequency second mode and modulate it with the 3D vortical first mode disturbance. There are fortunately many opportunities worth studying.

The objective of our present DNS study is to simulate interactions of the different modes of disturbances introduced at different frequencies, and find out what happens if they grow concurrently.

NUMERICAL TECHNIQUE

In our DNS we solve numerically the 3D Navier–Stokes equations for compressible gas. The numerical computations are performed with the time-explicit Navier–Stokes numerical code based on a 5th order WENO scheme of Jiang & Shu [3]. Diffusion terms are computed on a compact stencil with central-biased differences. The code is accurate in time due to 4th order Runge–Kutta algorithm. The code is parallelized via domain decomposition and MPI.

The simulations are performed with the assumption of the spatially evolving instability waves. We believe that this approach is much more physically realistic than the temporal simulation assuming periodicity along *X*. Boundary conditions specify at inflow the self-similar laminar basic flow at a given Reynolds number Re with superimposed time-dependent fluctuations. Linear stability eigenfunctions of the unstable disturbances are used for the inflow forcing. Numerical simulation is typically performed with one most unstable two-dimensional wave of the second mode with a given frequency ω and two symmetrical first mode instability waves propagating at angles χ and $-\chi$ to the basic flow. The specified frequency of the 3D first mode instability waves was $\omega/2$. The computational domain was long enough in *X* direction so that the low growth rate disturbances have enough length to evolve. Sponge layer at the far end of the domain ensured damping of the disturbances near outflow. We use periodic conditions in transverse direction. Computations were run at flow Mach number M =6, Reynolds number based on Blasius thickness of the boundary layer at inflow boundary Re=1000, wall temperature ratio to the free-stream static temperature Tw/Te=7. Initial amplitudes of the disturbances of the first and second mode were chosen both as A=0.005. Angle of 3D first mode wave propagation obtained from linear theory was χ=57°.

In our simulations we used grids condensed just above the plate and also resolving the critical layer, which is at 16-18 Blasius thicknesses above the plate. The entire mesh used herein was about 30 million grid cells. Computations were run at a distributed cluster using up to 64 CPU cores.

RESULTS

The results of our DNS show that, in accordance with linear stability theory, 2D disturbances of the second mode are dominant at initial stages and rapidly grow in *X*. The effect of the first mode disturbances at linear stage is basically a slight modulation of the second mode and also the presence of some transverse fluctuation components. At later stages, the second mode stabilizes, and the transverse fluctuations imposed by the first mode become visible (see lower left part of the Figure). They appear as 3D deformations of initially 2D rolls generated by the growth or the second mode. Here, the first mode is still unstable according to our linear stability calculations. Consequently, the 3D deformations of the rolls increase along *X*. The flow that formed after the rapid growth of the two-dimensional second mode now evolves in both *X* and *Z* directions being pumped by the energy of the first mode fluctuations. Soon, the formed 3D structures have no room to grow in *Z* and, consequently, form the strange worm-like λ structures protruding up from the boundary layer. Not surprisingly, these structures very much resemble those obtained in temporal simulations [4] of boundary layer transition at Mach 4.5 where they used periodic conditions in *X* and *Z*, and force random noise in *Z* direction.

Later on, we observe some complex interaction patterns and, finally, the onset of turbulence, which is manifested in rapid change of mean flow characteristics: velocity profiles, shear stress, etc.

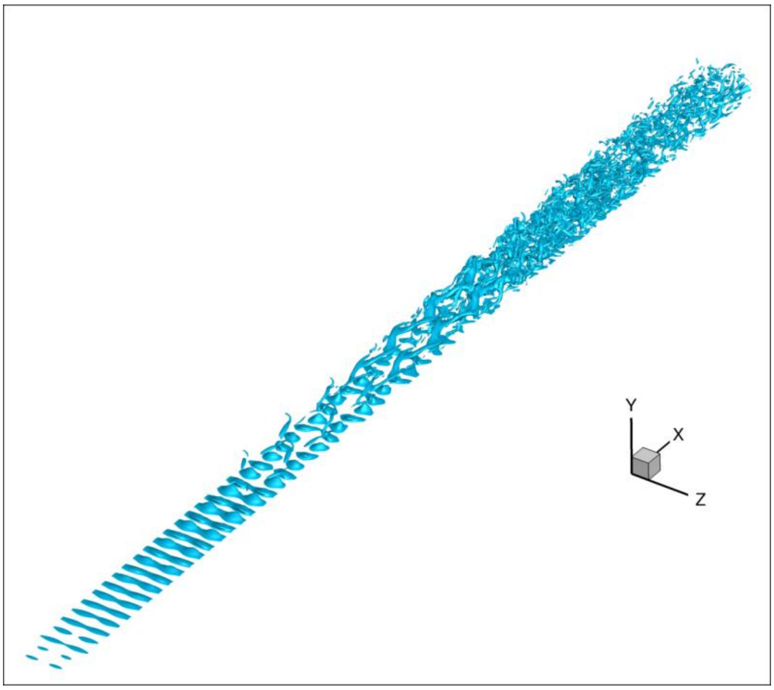

Figure. Non-linear interaction of the disturbances of planar acoustic mode and 3d vortical mode. Onset of laminar-turbulent transition is observed in the upper right part of the image. Q criterion surface.

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References

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