TANGLING CLUSTERING INSTABILITY FOR SMALL PARTICLES IN TEMPERATURE STRATIFIED TURBULENCE

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Abstract Particle clustering in a temperature stratified turbulence with small but still finite correlation time is studied. It is shown that the temperature stratified turbulence strongly increases the degree of compressibility of particle velocity field. This results in the strong decrease of the threshold for the excitation of the tangling clustering instability even for small particles. The tangling clustering instability in the temperature stratified turbulence is considerably different from the inertial clustering instability that occurs in non-stratified isotropic and homogeneous turbulence. While the inertial clustering instability is caused by the centrifugal effect of the turbulent eddies, the mechanism of the tangling clustering instability is related to the temperature fluctuations generated by the tangling of the mean temperature gradient by the velocity fluctuations. The temperature fluctuations produce the pressure fluctuations and cause particle clustering in regions with increased pressure fluctuations. The ratio of growth rates of the tangling and inertial clustering instabilities is proportional to $\sqrt{\mathrm{Re}}\,(\ell_0/L_T)^2/(3\mathrm{Ma})^4$, where Re is the Reynolds number, Ma is the Mach number, ℓ_0 is the integral turbulence scale and L_T is the characteristic scale of the mean temperature variations. It was found that depending on the parameters of the turbulence and the mean temperature gradient there is a preferential particle size at which the particle clustering due to the tangling clustering instability is more effective. The tangling clustering instability may be effective, e.g., in atmospheric turbulence with temperature inversions.

1. Inertial particle clustering and inertial clustering instability

Formation of spatial inhomogeneities in the number density distribution of small inertial particles in a turbulent flow (so called particle clustering) has attracted considerable attention in the past decades. The enhanced number density of particles within the cluster may affect the particle interactions, their dynamics and collisions, coalescence etc. The dynamics of particle collisions is relevant to many phenomena in nature such as the raindrop formation, atmospheric aerosols dynamics, and numerous industrial processes.

The clustering of inertial particles in a turbulent flow may be caused by the centrifugal effect, which implies that the inertial particles are locally accumulated in regions between the turbulent eddies. These regions have a low vorticity, high strain rate, and maximum fluid pressure. Therefore, turbulent vortices act as small centrifuges that push heavy particles to the boundary regions between the eddies by the inertial forces creating concentration inhomogeneities. This effect is known as the inertial particle clustering.

There are two scenarios of particle clustering: (i) the "source" clustering related to the source term in the equation for fluctuations of the particle number density; and (2) the clustering caused by the clustering instability. Most of analytical studies were related to the "source" inertial clustering. Another possible scenario of the particle clustering is a consequence of a spontaneous breakdown of their homogeneous spatial distribution due to the clustering instability. The inertial clustering instability has been studied in [1]-[3].

2. Particles in stably stratified turbulence

The particle clustering in stably stratified turbulence is essentially different from the inertial clustering in non-stratified isotropic and homogeneous turbulence. One of the important effects in the temperature stratified turbulence is turbulent thermal diffusion [4] whereby the inertial particles are accumulated in the vicinity of the temperature minimum of the surrounding fluid and a nonzero gradient of the mean particle number density is formed. Turbulent thermal diffusion is a purely collective phenomenon caused by temperature stratified turbulence and resulting in a pumping effect, i.e., the appearance of a non-zero mean effective velocity of particles in the direction opposite to the mean temperature gradient. A competition between the turbulent thermal diffusion and turbulent diffusion determines the conditions for the formation of large-scale particle clusters in the vicinity of the mean temperature minimum.

Particle clustering in the temperature stratified turbulence can be more effective than the inertial particle clustering in isothermal turbulence. [5] Tangling of the mean temperature gradient by the velocity fluctuations generates the temperature fluctuations. The fluctuations of fluid temperature and velocity are tangled and correlated with the fluctuations of fluid pressure. Therefore, the pressure fluctuations increase fluctuations of the particles number density and essentially increase the rate of formation of the particle clusters. Moreover, tangling of the mean gradient of particle number density (formed by the turbulent thermal diffusion) generates additional fluctuations of particle concentrations and contributes to the particle clustering. The steady-state regime of the tangling clustering without instabilities in temperature stratified turbulence (so called the "source" tangling clustering) has been recently studied both experimentally and theoretically in [5].

3. Tangling clustering instability

Based on the theoretical analysis, we demonstrated [6] the strong influence of the temperature fluctuations on the particle clustering. These temperature fluctuations, caused by the tangling of the mean temperature gradient by the velocity fluctuations, produce the pressure fluctuations and enhance considerably the particle clustering, so that the growth rate of the tangling clustering instability is $\sqrt{\mathrm{Re}}\,(\ell_0/L_T)^2/(3\mathrm{Ma})^4$, times larger than the growth rate of the pure inertial clustering instability.

We found that the temperature fluctuations strongly increase the degree of compressibility of particle velocity field in a temperature stratified turbulence:

$$\sigma_{\rm v} \equiv \frac{\langle(\boldsymbol{\nabla}\cdot\boldsymbol{v})^2\rangle}{\langle(\boldsymbol{\nabla}\times\boldsymbol{v})^2\rangle} = \frac{1}{3}\,\mathrm{St}^2\,\frac{\mathrm{Re}^{1/2}}{\mathrm{Ma}^4}\left|\frac{\ell_0\boldsymbol{\nabla}T}{T}\right|^2 .$$

We demonstrated that the growth rate of the tangling clustering instability and the particle number density inside the cluster after saturation of this instability depends on the parameter $\Gamma \approx \left(\mathrm{Re}^{1/4}/9\,\mathrm{Ma}^2\right)\,\ell_0\,|\boldsymbol{\nabla}T|\,/T$.

The exponential growth at the linear stage of the tangling clustering instability is saturated by nonlinear effects. For small particles one of the mechanism of saturation of the growth of the tangling clustering instability is exhaustion of the particles in the surrounding area. Indeed, the tangling clustering instability causes strong redistribution of particles so that inside the clusters the particle number density is strongly increased at some instant, while in surrounding regions it is essentially decreased. Note also that with a significant decrease in the number density of particles the hydrodynamic description becomes not applicable. The resulting estimate for the maximum number density of particles inside the cluster: $n_p^{\max}/N = \left(1+\lambda\,\mathrm{Sc}^{\lambda/2}\,\ln\mathrm{Sc}\right)^{1/2}$, where $\mathrm{Sc}=\nu/D$ is the Schmidt number, ν is the kinematic viscosity and D is the coefficient of the Brownian diffusion of particles. The parameter λ changed from $1/2$ to $5/2$ depending on the parameter Γ and size of particles.

In general, the other nonlinear mechanisms can also affect and limit the maximum particle concentration, the value of $n_p^{\max}/N$, achieved within the cluster. For example, a momentum coupling of particles and turbulent fluid is essential when the mass loading parameter $\phi = m_{\rm p} n_{\max}/\rho$ is of the order of unity. This condition yields: $n_p^{\max}/N \sim (\phi/4a_p^3 N)(\rho_p/\rho)^{-1}$, where ρ_p is the material density of spherical particles. For solid particles $\rho_p/\rho = 10^3$ and $N = 10^3$ cm^{-3}, we obtain that for $a_p = 1\mu$m, the ration $n_p^{\max}/N \sim 10^6$, for $a_p = 3\mu$m, the ration $n_p^{\max}/N \sim 10^4$, while for $a_p = 10\mu$m, the ration $n_p^{\max}/N \sim 250$. Therefore, the effect of the dynamical coupling should be taken into account for $a_p > 3\mu$m together with the mechanism caused by the exhaustion of the particles in the surrounding area of the cluster.

For small enough particles the tangling clustering instability in the presence of the imposed mean temperature gradient may result in the formation of small-scale clusters with the number density of particles exceeding the ambient average particle number density by several orders of magnitude. The tangling clustering instability in the presence of temperature gradient may enhance significantly the collision rate of small particles, which is of interest for atmospheric physics and many other practical applications. In particular this effect can substantially accelerate the coalescence of small droplets.

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