EXPERIMENTAL SCALAR SPECTRA IN CHAOTIC ADVECTION

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Abstract We investigate scalar spectra in a Hele-Shaw cell, where mixing is achieved through chaotic advection. Even though the flow is almost 2D (the dependance in z is parabolic velocity profile), we recover features found numerically in 3D chaotic advection, that are also similar with what one would expect in a turbulent flow: a -1.1 power-law behavior is found at intermediate scales, while the spectrum decays exponentially at the smallest scales at which diffusion is effective. Moreover, a scaling in $Pe^{-1/2}$ (therefore in $Sc^{-1/2}$) is found, which is consistent with the scaling of the Batchelor scalar dissipation scale.

INTRODUCTION

Measuring the Batchelor scalar scale at high Péclet (or Schmidt) number in a turbulent flow is experimentally almost out of reach since it is much smaller than the Kolmogorov velocity scale. It has been suggested that the mechanism that brings large scalar scales down to the Batchelor dissipation scale in chaotic advection may be interpreted as a manifestation of Lagrangian random strain, consistent with the cascade from the Kolmogorov scale down to the Batchelor scale in turbulent flows [4]. We investigate here experimentally scalar spectra obtained with a periodic chaotic flow in a Hele-Shaw cell, and expect to recover a behaviour similar with what is expected in a turbulent flow at wavenumbers higher then $\frac{2\pi}{\eta}$.

EXPERIMENTAL APPARATUS

Mixing is achieved using a Hele-Shaw cell (Figure 1a) of dimensions 50 $\times$ 50 $\times$ 4 mm^3. Chaotic advection is obtained using a time-periodic protocol [5] illustrated in Figure 1b. Alternated crossed injections are carried out with two peristaltic pumps. PLIF method is used to follow scalar evolution consisting in fluorecein dye or colloids labeled with fluorescent dye, introduced at $t = 0$ inside the Hele-Shaw cell.

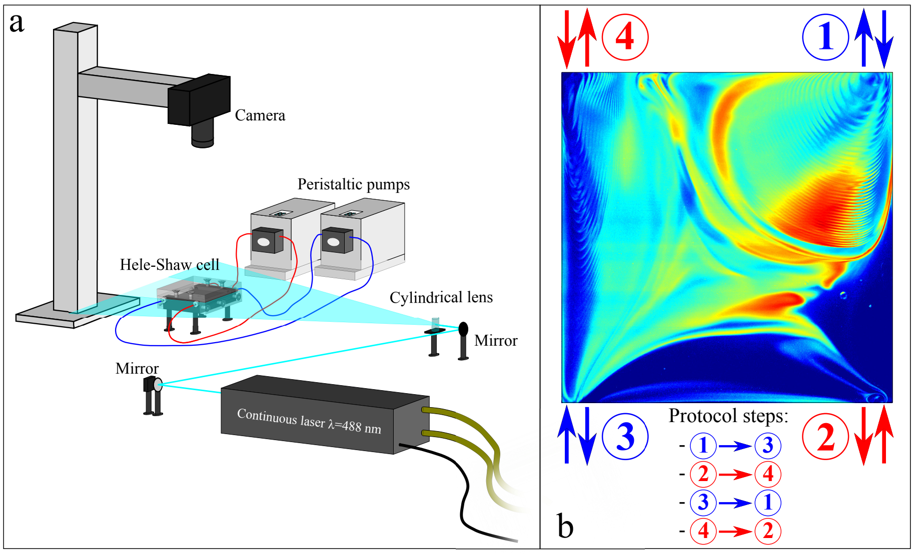

Figure 1. a: Sketch of the experimental setup. b: Time-periodic protocol and example of dye visualization.

RESULTS AND DISCUSSIONS

The scalar spectra are obtained thanks to a two-dimensional discrete Fourier transform, corrected with Hanning window method. The characteristics of mixing depend on the Péclet number:

$$Pe = \frac{LU}{D} \tag{1}$$

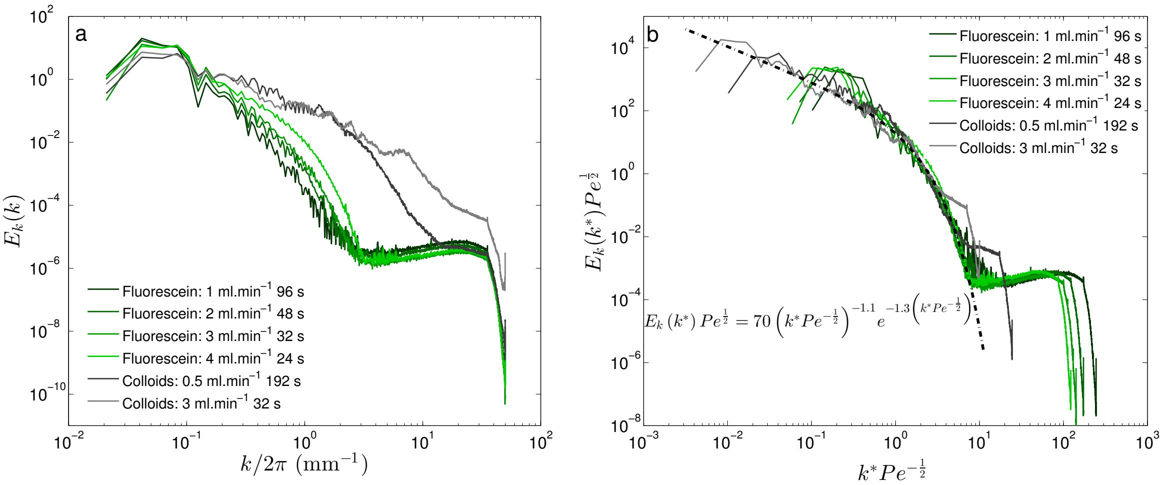

Figure 2. a: Comparison of different spectra. b: Rescaled spectra $E_k\left(k^*\right)Pe^{\frac{1}{2}}$ plotted as a function of $k^*Pe^{-\frac{1}{2}}$.

where L is the characteristic length of the Hele-Shaw cell (50 mm), U the mean flow velocity and D the mass diffusion coefficient. In our experiment, the Péclet number is varied from 1.10^4 to 6.10^6 whether by considering different flow rates (associated with different periods so as to keep the dimensionless pulse volume constant [1]), or different scalars (therefore different diffusion coefficients). The resulting spectra are given in Figure 2a. We introduce dimensionless wavenumber $k^* = kL$, and we are able to collapse the experimental spectra by plotting $E_k\left(k^*\right)Pe^{\frac{1}{2}}$ as a function of $k^*Pe^{-1/2}$ (Figure 2b). Although our flow is quasi-bidimensional, we recover the same behavior as in Toussaint *et al.* [7] for their numerical spectra of 3D chaotic advection. We obtain indeed the following scaling:

$$E_k\left(k^*\right)Pe^{\frac{1}{2}} = 70\left(k^*Pe^{-\frac{1}{2}}\right)^{-1.1}e^{-1.3\left(k^*Pe^{-\frac{1}{2}}\right)} \tag{2}$$

At intermediate scales, the behavior is a -1.1 power-law, consistent with the Kraichnan theory [2] for a decaying scalar (the -1 power-law corresponds to the stationary case). At smaller scales (larger k^*), we recover an exponential behavior, which Pierrehumbert [3] explained by the existence of a strange eigenfunction of the scalar field [5]. Finally, the dependance in $Pe^{-1/2}$ (therefore in $Sc^{-1/2}$) is consistent with the scaling of the Batchelor scalar dissipation scale.

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