ELASTIC ENERGY TRANSFER BY FLEXIBLE POLYMERS IN FLUID TURBULENCE

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Abstract We present a study of the energy transfer in the bulk of a turbulent flow with dilute long-chain polymer additives. Based on prior work by de Gennes and Tabor [1, 2] we introduce a theory that balances the energy flux through the turbulent cascade with that of the energy flux into the elastic degrees of freedom of the dilute polymer solution. We propose a refined elastic length scale, r_ε, which describes the effect of polymer elasticity on the turbulence energy cascade. Our experimental results agree excellently with this new energy flux balance theory.

INTRODUCTION

In fully developed three dimensional fluid turbulence the fluctuating energy is supplied at large scales and dissipated by viscosity at small scales, while the energy cascades through the intermediate inertial range scales without loss. One very important question is how this cascade process is altered, when an additional agent that can also transport energy, such as long-chain polymers, is added to the fluid. To date, the only relevant theory on the interaction between polymers and turbulence cascade is the "energy balance theory" by de Gennes and Tabor [1, 2]. It states that the turbulence energy cascade is essentially unaltered down to a scale at which the energy stored in the elastic degrees of freedom of the polymer is equal to the kinetic energy of the flow. So far experiments on fully developed three dimensional turbulence do not convincingly support this theory [3]. Here we argue that this may not be surprising as it is the turbulent flux of energy and not the energy itself that determines the inertial range properties of turbulence.

Here we propose what may be called an "energy flux balance theory". In this theory, the turbulent energy flux through the cascade (or the turbulent energy transfer rate from scale to scale) is gradually reduced by the energy transfer through stretching and recoiling of the polymer chains, with the elastic energy flux becoming dominant below a scale that is given by the balance of elastic and turbulent energy flux. This energy flux balance theory is supported quantitatively by our experimental data measured over a wide range of parameters in fully developed turbulence.

ENERGY-FLUX BALANCE THEORY

To consider the interaction between flexible long-chain polymers and the dissolving fluid, one may regard a single polymer chain as an entropic spring that is constantly stretching and coiling back in the flow [4, 1, 2] . The turbulence fluctuations at different scales contribute unequally to the stretching of the polymer chain. In particular, Lumley concluded that only those fluctuations with time scale $\tau_r \lesssim \tau_p$ can stretch the polymer chain [5], where τ_p is the entropic viscous relaxation time of the polymer chain. This "time criterion" defines the Lumley scale $r^* \equiv (\varepsilon {\tau_p}^3)^{1/2}$ (see e.g. [6]), whose physical meaning is that below this scale the local fluid deformation would be strong enough to stretch polymers. The Lumley scale, on the other hand, cannot answer how polymers dissolved at a certain concentration will affect the flow. This was addressed by the "energy balance theory" proposed by Tabor & de Gennes [1]. In that theory, the polymer elastic energy per volume is $E_e(r) \sim c_p kT(r^*/r)^{5n/2}$ for $r \ll r^*$, where c_p is the number of polymer chains per unit volume, k is the Boltzmann constant, T is the temperature of the fluid, and n is an unknown exponent that is related to the average stretching dimensions of the local flow field. The turbulent energy cascade will be truncated below a scale $r^{**} = (kT\rho^{-1}{c_p}^{-1}{\varepsilon_T}^{\frac{5n}{4}-\frac{2}{3}}{\tau_p}^{\frac{15n}{4}})^{\frac{6}{15n+4}}$ at which the polymer elastic energy balances the kinetic energy of the turbulent fluctuations, where the polymer relaxation time $\tau_p \approx (N^{3/5}a)^3\mu/kT$, with N being the number of monomers per chain, a the length of a monomer, and μ the dynamic viscosity of the fluid.

Note that the energy-balance theory assumes that the turbulence energy cascade is unaffected at scales $r > r^{**}$, which is, however, not consistent with the theory itself as polymers already gain elastic energy from the stretching by eddies of size $r^{**} < r < r^*$ and hence must have diverted part of the turbulence energy flux at scales $r > r^{**}$. Assuming the time scale for polymers to transfer elastic energy down scales is τ_p, then the elastic energy flux is $\varepsilon_e(r) \sim E_e(r)/(\tau_p\rho) \sim \frac{kTc_p}{\tau_p\rho}(\frac{r^*}{r})^{5n/2}$, which increases as r decreases and can dominate the turbulence energy flux ε_T at a new scale r_ε that is determined by the balance of the two fluxes $r_\varepsilon = (AkT/\rho)^{\frac{2}{5n}}{c_p}^{\frac{2}{5n}}{\varepsilon_T}^{\frac{1}{2}-\frac{2}{5n}}{\tau_p}^{\frac{3}{2}-\frac{2}{5n}}$, where A is a proportional factor. For $r_\varepsilon < r < r^*$, the turbulent energy flux is only slightly affected and inertial range scaling of the cascade would still capture the behavior to leading order. However, for $r < r_\varepsilon$, the turbulent energy transfer will be strongly reduced.

COMPARISON WITH EXPERIMENTAL RESULTS

To test this new theory, we compare its predictions with particle tracking data of turbulent water flows with polyacrylamide (PAM, $M_W = 18 \times 10^6$) additives between counter-rotating baffled disks in a cylindrical tank, known as the von Kármán

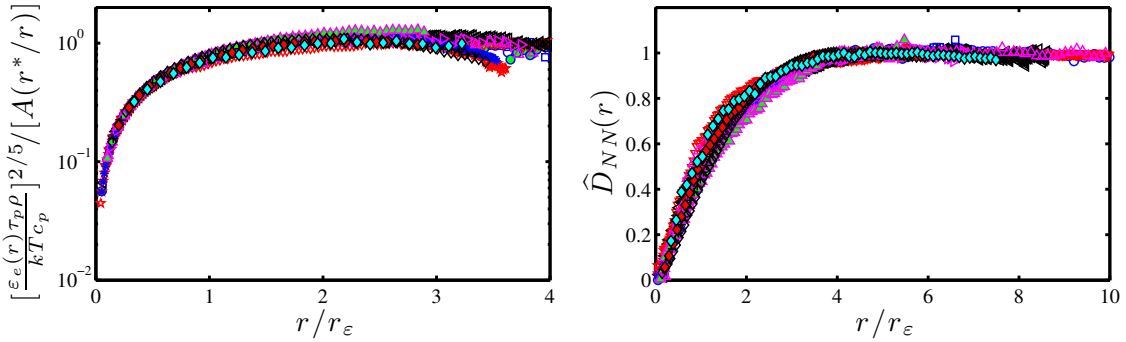

Figure 1. Collapsing of data from 14 experiments with wide ranges of Reynolds number R_λ, Weissenberg number Wi, and polymer concentration ϕ when the scale r is normalized by the "energy-flux balance scale" r_ε as predicted by the theory. (a) The elastic energy flux $\varepsilon_e(r)$; (b) The normalized velocity structure function $\widehat{D}_{NN}(r) \equiv \widetilde{D}_{NN}/\varepsilon_T$.

swirling flow. We first determine the exponent n and the constant A by fitting the elastic energy flux $\varepsilon_e(r)$ to the prediction $\left[\frac{\varepsilon_e(r)\tau_p\rho}{kTc_p}\right]^{2/5} = A(r^*/r)^n$, where $\varepsilon_e(r)$ is obtained from the difference between the compensated second order transverse velocity structure function $\widetilde{D}_{NN}(r) = \frac{1}{r}[\frac{3}{4}D_{NN}(r)/C_2]^{3/2}$ with and without polymers. Note that for Newtonian flow $\widetilde{D}_{NN}(r)$ in the inertial range gives the energy transfer rate ε_T. Data obtained from 14 different data sets with wide ranges of R_λ, Weissenberg number Wi, and polymer concentration ϕ all fit well with $n = 1.0$ and the corresponding A is within 101 ± 17 except the ones with small Wi, which are very like below a critical Wi for coil-stretch transition [7]. Note that $n = 1$ implies that polymers are most effectively stretched in locally biaxial extensional regions of the flow. This is consistent with the properties of velocity gradients in turbulent flows [8] and previous findings of polymer behavior in turbulence [9, 10]. This value is also close to the result from a previous compilation of drag-reduction data from different turbulent pipe flows [11]. With known n and A, the scale r_ε is given as $r_\varepsilon = A(kT/\rho)^{0.4}{c_p}^{0.4}{\varepsilon_T}^{0.1}{\tau_p}^{1.1}$, which then can be used to collapse all the measured elastic energy flux $\varepsilon_e(r)$. In Figure 1(a) we plot $[\frac{\varepsilon_e(r)\tau_p\rho}{kTc_p}]^{\frac{2}{5}}/[A(r^*/r)^{1.0}]$ against r/r_ε. All the 14 data sets collapse and show a plateau when $r \gtrsim r_\varepsilon$, which is the expected scaling range for elastic energy flux. For $r < r_\varepsilon$ polymer elasticity dominates the energy transfer process, and the constant energy cascade does not hold any more. Thus below those scales the elastic energy transfer rate could not be obtained by simply subtracting $\widetilde{D}_{NN}(r)$ for $\phi > 0$ from the $\phi = 0$ case. Furthermore, Figure 1(b) shows $\widehat{D}_{NN}(r) \equiv \widetilde{D}_{NN}/\varepsilon_T$ against r/r_ε. All the datasets collapse and the plateau in $\widehat{D}_{NN}(r)$ appears at $r/r_\varepsilon \gtrapprox 3$, in good agreement with the theory.

The physical meaning of r_ε is similar to that of the Kolmogorov scale η: Both correspond to a scale at which the inertial range turbulent energy cascade is truncated by a mechanism whose effect is negligible in the inertial range but increases at small scales. On the other hand, for polymer solutions, at scales $r < r_\varepsilon$ there is still room for interesting dynamics. It has been shown that in a smooth velocity field the elastic instability can leads to elastic turbulence [12]. Our theory suggests that in turbulence this might occur when the polymer stress dominates the fluid stress, *i.e.*, at scales below r^{**}.

We are grateful to the Max Planck Society and the Deutsche Forschungsgemeinschaft (through grant XU91-3) for financial support. H.-D. Xi also thanks the Alexander von Humboldt Foundation for a generous fellowship.

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