RELATIVE VELOCITIES OF INERTIAL PARTICLES IN RANDOM FLOWS

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Abstract We compute the distribution of relative velocities for a model of heavy particles suspended in a turbulent flow, quantifying the caustic contribution to the moments of relative velocities. Our conclusions are in excellent agreement with numerical simulations of particles suspended in randomly mixing flows, and in quantitative agreement with published data on direct numerical simulations of particles in turbulent flows.

Collision velocities of particles suspended in randomly mixing or turbulent flows ('turbulent aerosols') have been studied intensively for several decades. This is an important topic because the stability of turbulent aerosols is determined by collisions between the suspended particles. Direct numerical simulations of particles in turbulent flows [6, 7] show that collision velocities increase precipitously as the particle damping rate is decreased beyond a threshold value which depends on the nature of the flow. In [8] (see also [3]) this steep increase of collision velocities was attributed to the fact that singularities in the particle dynamics result in large relative velocities at small separations. These singularities occur when phase-space manifolds (describing the dependence of particle velocity upon particle position) fold over. In the fold region, the velocity field at a given point in space becomes multi-valued, giving rise to finite velocity differences between particles at the same position in configuration space. This is in contrast to the relative velocity between particles in a single-valued smooth particle-velocity field, which tends to zero as the particles approach.

Numerical simulations show that heavy particles suspended in turbulent flows and random flows cluster onto a fractal, they are not smoothly distributed along phase-space manifolds. This raises the question: what effect does this have on the distribution of relative velocities at small separations in the wake of a singularity? Numerical simulations also show (see [1, 2, 4, 5]) that the distribution of large relative velocities at fixed relative separations exhibit a power-law scaling.

We introduce a new general principle explaining the numerically observed power laws in the distribution of relative velocities at small spatial separations. We compute the distribution of relative velocities and separations of inertial particles suspended in randomly mixing flows. Our results are obtained by matching asymptotically known forms of the distribution. The form of the computed distribution is characterised in terms of the phase-space fractal dimension D_2 together with up to two additional matching parameters. For a white-noise flow in one spatial dimension both the phase-space fractal dimension and the matching parameter can be calculated analytically. For general flows or in higher spatial dimensions the parameters must be obtained by numerical simulations.

The method clearly exhibits which aspects of the distribution are universal, that is independent of the statistical properties of the flow. At small separations R, for example, and not too large relative velocities $|V_R|$, the distribution of relative velocities exhibits a power-law form $\rho(V_R, R) \sim |V_R|^{D_2-d-1}$ provided $D_2 < d+1$, where d is the spatial dimension.

We find that the above-mentioned singularities (caustics) make a substantial contribution to relative velocities at small separations. Moreover, our result support the ansatz suggested in [8], that the collision rate is a sum of two terms, a smooth contribution and a singular contribution due to caustics, with different characteristic dependencies on the particle size.

References

[1] J. Bec, L. Biferale, M. Cencini, A. Lanotte, and F. Toschi. Intermittency in the velocity distribution of heavy particles in turbulence. *J. Fluid Mech.*, **646**:527–536, 2010.
[2] M. Cencini. talk MP0806 Cencini WG3 2009.pdf given at working group meeting of cost action mp0806. 2009.
[3] G. Falkovich, A. Fouxon, and G. Stepanov. Acceleration of rain initiation by cloud turbulence. *Nature*, **419**:151, 2002.
[4] K. Gustavsson and B. Mehlig. Distribution of relative velocities in turbulent aerosols. *Phys. Rev. E*, **84**:045304, 2011.
[5] M. Reeks K. Gustavsson, E. Meneguz and B. Mehlig. Inertial-particle dynamics in turbulent flows: caustics, concentration fluctuations and random uncorrelated motion. *New J. Phys.*, **14**:115017, 2012.
[6] S. .Sundaram and L. R. Collins. *J. Fluid. Mech.*, **335**:75, 1997.
[7] L. Wang, A. S. Wexler, and Y. Zhou. *J. Fluid Mech.*, **415**:117, 2000.
[8] M. Wilkinson, B. Mehlig, and V. Bezuglyy. Caustic activation of rain showers. *Phys. Rev. Lett.*, **97**:048501, 2006.