Energy and helicity spectra in the shell model with distributed helicity injection

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Abstract We present the results of modeling the development of homogeneous and isotropic turbulence with a source of helicity distributed in scale. We use the shell model for numerical simulation of the turbulence at high Reynolds number. Energy in our model is injected at certain large scale only whereas the source of helicity is distributed in scale and its characteristic scale may differ from the characteristic scale of energy source. The idea of our study related to the fact that quite often the helicity associated with the presence of coherent structures of different scales in the turbulent flow (e.g. atmospheric vortexes). The calculations are showed that the distribution helicity injection in scale (with constant injection of energy at the main scale) leads to a significant change in the behavior of the energy and helicity spectra. In particular, we found that, depending on the parameters of the injection helicity in the energy spectrum will be sufficiently extended interval with a slope of "-7 / 3" on a par with the Kolmogorov slope "-5/3". Thus, the proposed approach allows us to extend the shell model to study a broader class of turbulent flows.

MODEL

Currently, there are a large number of works in which there is a calculation of the spectra of energy *E(k)* and helicity *H(k)* of homogeneous and isotropic turbulence. Typically, injection of energy and helicity in these studies is on the main scale which leads to the Kolmogorov their behavior in the inertial interval. In this case the shell model is no exception, and leads to the same results [1]. In this paper addressed the case of helical turbulence with constant spectral flux and helicity has been shown that the energy spectrum is not changed and helicity transported to smaller scales as passive scalar. On the other hand, there is many experimental facts indicating that in the presence of a non-zero mean helicity the energy turbulence spectrum in the inertial interval has a steeper slope "-7 / 3" (see for example [2] and references therein).
In the present work we use H1-type shell model for numerical simulation of the turbulence at high Reynolds number (see for review [3]). Energy in our model is injected at certain large scale only whereas the source of helicity is distributed in scale and its characteristic scale may differ from the characteristic scale of energy source. The proposed external force *f* is given by the formulas

$$f_a(t,k) = \frac{\eta \frac{b(t)}{k} - \varepsilon a(t)}{b^2(t) - a^2(t)} \qquad \text{and} \qquad f_b(t,k) = \frac{\varepsilon b(t) - \eta \frac{a(t)}{k}}{b^2(t) - a^2(t)} \qquad (1)$$

where $a(t)$ and $b(t)$ are real and imaginary parts of a shell variable at scale k. Energy pumping takes a place only in one shell with $n = 0$ ($k=1$) with $\varepsilon = 1$, while helicity pumping occurs across the spectrum according to the law $\eta = \eta_0 k^\alpha$ for $k>1$ where η_0 is a constant and $\alpha \le 0$. The idea of such forcing is to support highest possible level of helicity at given scale. It is know that value of helicity at given scale k is limited by admissibility condition $H(k) \le kE(k)$.

MAIN RESULT

The energy and helicity spectra for the force (1) with the parameters η_0=1, α=0 and α=-1 is resented in Fig.1. The calculations are shown that the distribution helicity injection in scale (with constant injection of energy at the main scale) leads to a significant change in the behavior of the energy and helicity spectra. In particular, we see that, depending on the parameters of the injection helicity in the energy spectrum will be sufficiently extended interval with a slope of "-7/3" on a par with the Kolmogorov slope "-5/3".

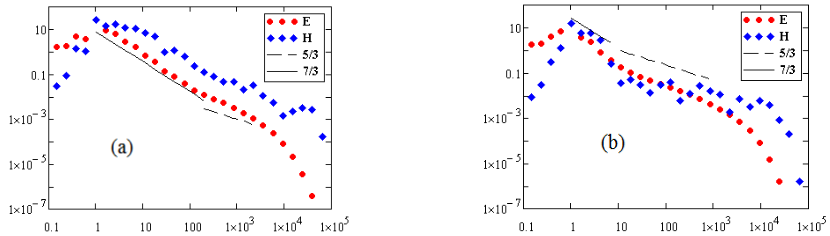

Figure 1. Energy and helicity spectra for $\mathrm{Re} = 10^5$. (a) α=0 and (b) α=-1.

References

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