ULTIMATE RAYLEIGH-BÉNARD AND TAYLOR-COUETTE TURBULENCE

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Abstract We report on recent experimental and numerical findings of ultimate turbulence for the Rayleigh-Bénard (RB) and the Taylor-Couette (TC) system, highlightening the analogy between these two systems. However, in the TC system the onset of the ultimate regime is much earlier than in the RB system as the mechanical forcing in TC is more efficient than the thermal forcing in RB. We also show and explain the logarithmic boundary layer profiles for both systems in the ultimate regime.

The two classical flows in closed systems are Rayleigh-Bénard (RB) flow – the flow in a closed box heated from below and cooled from above – and Taylor-Couette (TC) flow – the flow in between two independently rotating coaxial cylinders. These two systems had always been the most popular playgrounds for the development of new concepts in physics of fluids, be it instabilities, nonlinear dynamics and chaos, pattern formation, or turbulence [1–4]. The reasons why these systems are so popular are from our point of view: (i) These systems are mathematically well-defined by the (extended) Navier-Stokes equations with their respective boundary conditions; (ii) for these closed system exact global balance relations between the respective driving and the dissipation can be derived; and (iii) they are experimentally accessible with high precision, thanks to the simple geometries and high symmetries. (iv) The boundaries and resulting boundary layers play a prominent role in both systems and RB and TC turbulence are thus ideal systems to study the interaction between boundary layers and bulk.

RB and TC flow not only share the property of being closed-system flows with exact balance equations, but enjoy a much deeper formal analogy based on the underlying Navier-Stokes equations. At onset of instabilities it had first been discovered in 1969 [5], and then extended to the fully turbulent regime [6–8]. Our recent experimental findings [9–11] seem to confirm this analogy. In this contribution we want to further elucidate this analogy between these two "twins of turbulence" [12].

For both RB and TC flow, the most remarkable state is the "ultimate turbulence" state, for which we recently found strong evidence both for RB turbulence [11] and for TC turbulence [10]. This state is distinguished by optimal transfer properties of the system – heat transfer in RB and angular momentum transfer in TC. We believe that in this ultimate turbulent state the laminar-type boundary layers have become unstable through a (nonlinear) shear flow instability, giving way for turbulent boundary layers which allow for much more efficient transfer properties [20–22]. Inside the turbulent boundary layers, tiny viscous and thermal sublayers must control the heat transfer or angular velocity transfer in the whole domain. Boundary layers and bulk thus interact totally differently in this ultimate regime than in the classical RB or TC regimes.

The ultimate regime of turbulence sets in once the shear Reynolds number is beyond a certain threshold in the range between 200 and 400 [23]. As the mechanical driving is much more efficient in TC than in RB, this onset happens already at a Taylor number $Ta \sim 5 \cdot 10^8$, but only at a Rayleigh number $Ra \sim 10^{14}$. Beyond onset, the scaling of the dimensionless angular velocity transfer and heat transfer is $Nu \sim Ra^{0.38}$ and $Nu_\omega \sim Ta^{0.38}$, respectively, see figure 1a and 1b. In that ultimate regime the wind Reynolds numbers Re_w scale as $\sim Ra^{1/2}$ and $\sim Ta^{1/2}$, respectively, see figure 1c and 1d, in coherence with the prediction of Grossmann and Lohse [20], but in conflict with Kraichnan's prediction [21] which suggests logarithmic correction also for the wind velocity.

In both RB and TC flow in the ultimate regime the transported quantity develops a logarithmic profile, namely a logarithmic temperature profiles in RB [24] and a logarithmic angular velocity profile in TC. These profiles can theoretically be accounted for in the spirit of a turbulent Prandlt - von Karman type boundary layer [22].

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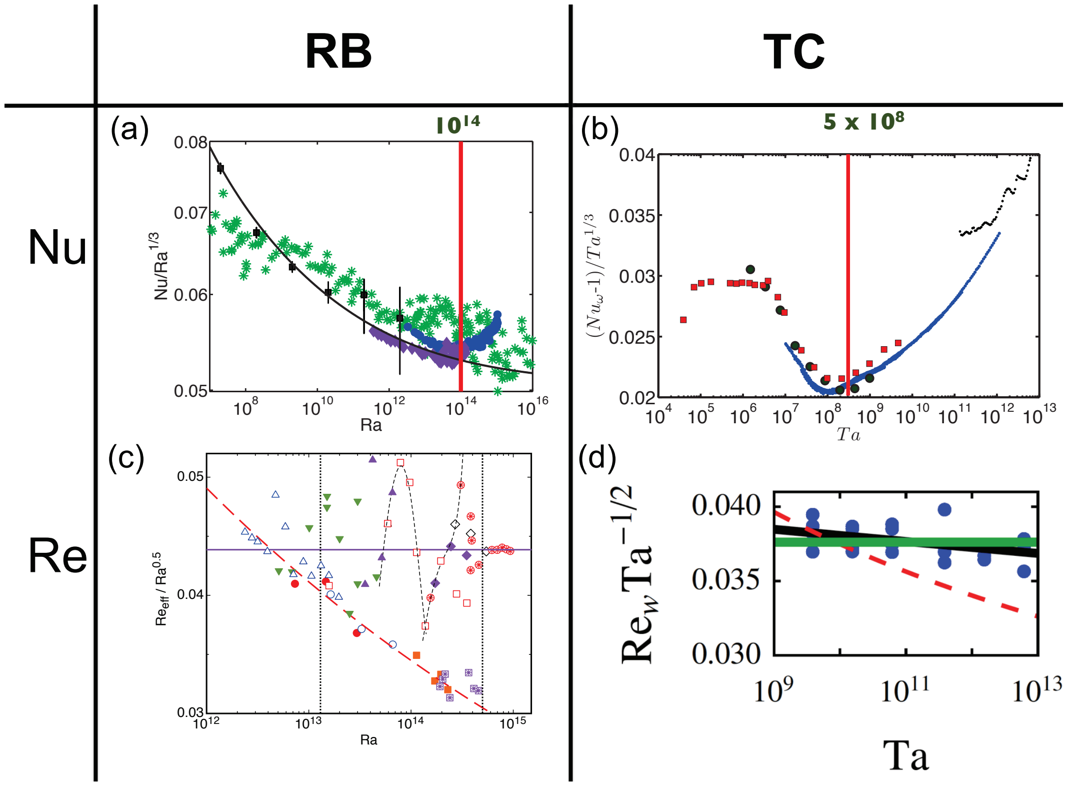

Figure 1. Scaling for the Nusselt number and the wind Reynolds number for RB and TC from various experiments and numerical simulations, showing the transitions to the ultimate regime (vertical red lines). (a) Nu vs Ra for $Pr = 0.7$ in a $\Gamma = 0.5$ sample. The data are from [13] (green stars), [14] (purple diamonds, $\Gamma = 1$), [15] (blue dots) and [16] (black squares, DNS). The black solid line is the GL-theory prediction. (b) Nu_ω vs Ta for $\eta = 0.714$. Data are from experiments (blue dots from [17] and black dots from [18]) and numerical simulations (green circles from [19] and red squares from present simulations in Twente). (c) Figure from He *et al.* [11]: compensated Reynolds number vs. Ra in turbulent RB flow. After a transition region for $10^{13} \leq Ra \leq 5 \times 10^{14}$, Reynolds number scales with $Ra^{1/2}$, which is in agreement with the results of Grossmann and Lohse [20]. (d) Figure from Huisman *et al.* [10]: compensated wind Reynolds number vs. Ta in turbulent TC flow. The solid dots are the measured data, and the black line is the best fit $Re_w = 0.0424 Ta^{0.495 \pm 0.010}$. The (red) dashed line is the Kraichnan prediction [21], and the horizontal (green) line is the prediction of Grossmann & Lohse [20].

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