SCALINGS OF THE OUTER ENERGY SOURCE OF WALL-TURBULENCE

A. Cimarelli[1], E. De Angelis[1], P. Schlatter[2], G. Brethouwer[2], A. Talamelli[1] & C.M. Casciola [3]
[1]*Dipartimento di Ingegneria Industriale, Università di Bologna, 47121 Forlì, Italy*
[2]*Linné FLOW Centre and Swedish e-Science Research Centre (SeRC), KTH Mechanics, Sweden*
[3]*Dipartimento di Ingegneria Meccanica e Aerospaziale, Università di Roma La Sapienza, 00185, Italy*

Abstract By means of the multidimensional description given by the Kolmogorov equation we study the energy transfer physics and the production mechanisms of wall-turbulent flows at moderately high Reynolds numbers. Two driving mechanisms are identified for the energy fluxes. The first stronger one, here called driving scale-range (DSR), belongs to the near-wall cycle. As expected, its topology remains unaltered with Reynolds number while its intensity is found to slightly increase with Re. The second mechanism, here called outer scale-range (OSR), takes place in the overlap layer and highlights different features in agreement with the attached eddies hypothesis usually considered to describe the overlap dynamics.

THE OVERLAP LAYER IN WALL TURBULENCE

Thanks to the inhomogeneity of the flow, the problem of wall-turbulent flows has been classically studied by dividing the flow domain into well characterized regions depending on wall-distance. In particular, wall-bounded flows are divided in a near-wall, inner region, and an outer region populated by large turbulent fluctuations. These two distinct regions are present in all wall-bounded flows and are coupled through an overlap region. While in the outer flow the velocity profile depends on the particular flow configuration, in the inner and overlap region exhibit a large degree of universality starting linearly from the wall and then approaching a logarithmic behaviour. These behaviours opportunely scaled with viscous units should collapse for different flows and different Reynolds numbers. The same scaling should be verified also for the turbulent intensity profiles and for all the statistical observables of the inner region. But, in fact, the near wall quantities exhibit a Reynolds dependence. For increasing Reynolds number the energy of the long turbulent fluctuations of the overlap layer becomes increasingly large. These large scale motion is not passive at all Reynolds number but is found to actively modulate the near-wall quantities at increasing Reynolds number [**?**]. An important consequence of the Reynolds number dependence of the overlap layer mechanisms is that the major contribution to the bulk turbulence should asymptotically come from the turbulent production in this region [7]. This aspect would lead to a high Reynolds number asymptotic state of wall turbulent flows which is dominated by an outer self-sustained mechanisms of this region. Hence the study of this layer assumes a crucial role in the understanding and modelling of wall-turbulent flows.

Actually, this asymptotic state has been addressed in the past and was the argument of one of the first theories developed in wall turbulence [8]. This theory is based on two main assumptions. The first one is that the overlap layer is populated by attached eddies, i.e. an organized flow pattern attached to the wall. The second is the equilibrium between production and dissipation, i.e. $\epsilon = \pi = u_\tau^3/ky$. According to this topology of the flow, the main features of wall turbulence can be described by the superposition of such eddies of different sizes and many predictions can be made and verified.

THE KOLMOGOROV EQUATION AND SOME RESULTS

An promising approach to study the basic mechanisms of the outer cycle has been recently proposed by [3, 1]. In these works, the multi-dimensional description of turbulence given by the generalized Kolmogorov equation, [6], has been used and proven fundamental for the understanding of the wall-turbulent physics. Here, we want to extend this work by analysing how the turbulent energy is generated, transferred and dissipated among different scales and wall-distances in moderately high Reynolds number with particular attention to the outer self-regeneration mechanisms. To this aim we use a numerical data set consisting of three channel flow direct numerical simulation (DNS) at $Re_\tau = 550, 1000, 1500$ carried out with a pseudo-spectral approach. The domain and the resolution adopted are reported in table 1.

The Kolmogorov equation is the balance equation for the second order structure function, $\langle \delta u^2 \rangle$, where $\delta u^2 = \delta u_i \delta u_i$ and the fluctuating velocity increment at position X_s and vector separation r_s is $\delta u_i = u_i(X_s + r_s/2) - u_i(X_s - r_s/2)$. This equation, for the symmetries of the channel, is written in a four dimensional space (r_x, r_y, r_z, Y_c) and involves a

Case	Re_τ	L_x	L_y	L_z	$N_x \times N_y \times N_z$	Δx^+	Δz^+
DNS550	550	$8\pi h$	$2h$	$4\pi h$	$1024 \times 257 \times 1024$	13.5	6.7
DNS1000	1000	$8\pi h$	$2h$	$3\pi h$	$2560 \times 385 \times 1920$	9.8	3.7
DNS1500	1500	$12\pi h$	$2h$	$10.5h$	$6144 \times 577 \times 3456$	9.2	4.5

Table 1. Parameters of the channel flow simulations.

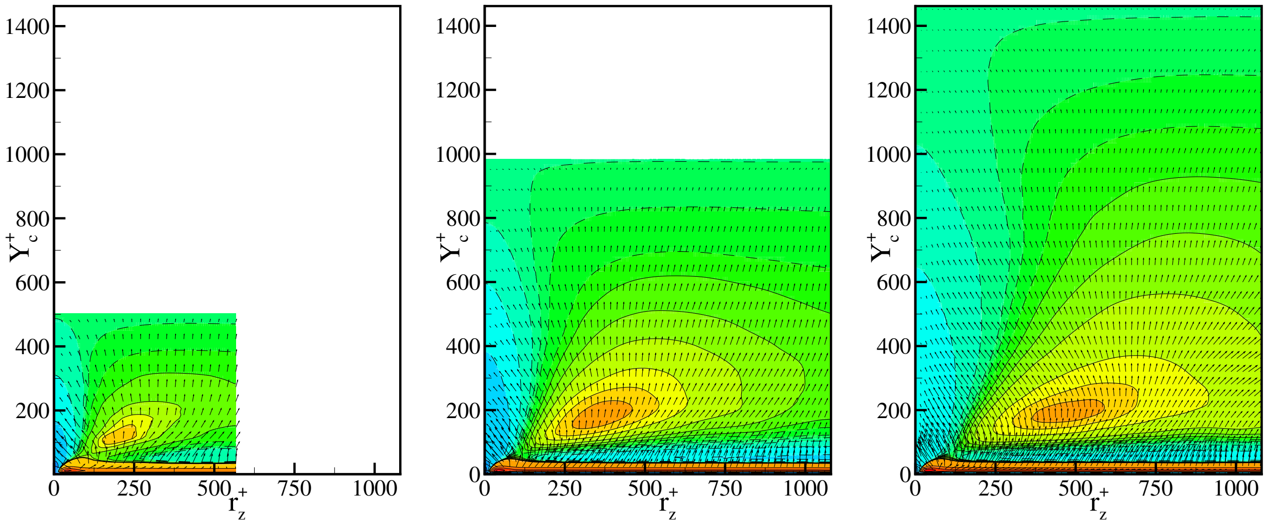

Figure 1. Channel flows, $Re_\tau = 550$, $Re_\tau = 1000$ and $Re_\tau = 1500$ from left to right. Energy source isolines in the (r_z, Y_c)-plane $(r_x = 0)$. Red colours encode the largest values of ξ. The vector field is the energy flux $\mathbf{\Phi}$.

four-dimensional vector field, $\mathbf{\Phi} = (\Phi_{r_x}, \Phi_{r_y}, \Phi_{r_z}, \Phi_c)$,

$$\nabla \cdot \mathbf{\Phi}(\mathbf{r}, Y_c) = \xi(\mathbf{r}, Y_c)\,, \tag{1}$$

where $\nabla\cdot$ is the four-dimensional divergence and $\xi = 2\langle \delta u \delta v \rangle \, (dU/dy)^* - 4\langle \epsilon^* \rangle$ is the energy source/sink. The energy flux in the space of scales is $\mathbf{\Phi}_r = (\Phi_{r_x}, \Phi_{r_y}, \Phi_{r_z}) = \langle \delta u^2 \delta \mathbf{u} \rangle - 2\nu \nabla_r \langle \delta u^2 \rangle$, while the spatial flux, a pseudo-scalar for the symmetries of the channel flow, is $\Phi_c = \langle v^* \delta u^2 \rangle + 2\langle \delta p \delta v \rangle/\rho - \nu/2 \partial \langle \delta u^2 \rangle / \partial Y_c$. Equation (1) highlights that the energy flux vector field is driven by the scalar field ξ. This energy source term reaches its maximum in a range of small scales well within the buffer layer, see figure 1. This region is the engine of wall-turbulence and, hence, will be called driving scale-range (DSR). However, a second peak in the energy source appears also further away from the wall, hereafter called outer scale-range (OSR), see figure 1. This outer peak of energy source, already observed in [3] and in [2] with LES data, belongs to the putative overlap layer and appears to be the result of a second outer turbulent production mechanism well separated from the near-wall dynamics. Though the OSR intensity is substantially smaller than for the DSR, its extent significantly increases with Re, actually supporting the conjecture of an outer mechanism.

In the final paper we will discuss, within the framework above delineated, different statistical scalings showing how the classical theories, based on the attached eddy hypothesis, work for the description and prediction of the overlap layer dynamics.

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