

ELASTIC-TURBULENCE-INDUCED MELTING OF A NONEQUILIRIUM VORTEX CRYSTAL IN A FORCED THIN FLUID FILM

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Abstract We perform a direct numerical simulation (DNS) of the forced, incompressible two-dimensional Navier-Stokes equation coupled with the FENE-P equations for the polymer-conformation tensor. The forcing is such that, without polymers and at low Reynolds numbers (Re), the film attains a steady state that is a square lattice of vortices and anti-vortices. We find that, as we increase the Weissenberg number (Wi), this lattice undergoes a series of nonequilibrium phase transitions, first to a distorted, steady crystals, then to a sequence of crystals that oscillate in time, periodically, at low Wi , and quasiperiodically, for slightly larger Wi . Finally, the system becomes disordered and displays spatiotemporal chaos and turbulence. We obtain the nonequilibrium phase diagram for this system in the $Wi - Re$ plane. We show that the Okubo-Weiss parameter Λ provides us with a natural measure for characterizing the phases and transitions in this diagram.

INTRODUCTION

Equilibrium melting transition, from a spatially periodic crystal to a homogeneous liquid has been studied extensively [1]. Nonequilibrium analogues of this transition have been explored in, e.g., the shear-induced melting of a colloidal crystals [1] and the turbulence-induced melting of a periodic array of vortices and anti-vortices in a forced, two-dimensional (2D), fluid film. We elucidate the nonequilibrium melting of a periodic array of vortices and anti-vortices in a forced, 2D, fluid film with polymer additives. We show that such an array can be melted either by increasing the Grashof (or Reynolds Re) number or by increasing the Weissenberg number Wi . In the former case, the disordered phase is a turbulent fluid that shows dissipation reduction, because of the polymer additives [2, 3]; in the latter case, the disordered state is a polymeric fluid that shows elastic turbulence (or rheochaos). We show that this system of vortices and anti-vortices also provides a way of studying the crossover from the dissipation-reduction to the elastic-turbulence regimes.

We solve the 2D, incompressible, Navier-Stokes (NS), with air-drag-induced friction, and the FENE-P equations for the polymer-conformation tensor \mathcal{C} in terms of the stream function ψ and the vorticity $\omega = \nabla \times \mathbf{u}(\mathbf{x}, t)$, where $\mathbf{u} \equiv (-\partial_y \psi, \partial_x \psi)$ is the fluid velocity at the point \mathbf{x} and time t , as follows (in the nondimensional form of Ref. [4]):

$$D_t \omega = \nabla^2 \omega / \Omega + \left(\frac{\nu^2 k^4}{F_{amp}^2} \right) \frac{\mu}{\tau_P} \nabla \times \nabla \cdot [f(r_P) \mathcal{C}] - \alpha \omega + F_\omega; \quad (1)$$

$$\nabla^2 \psi = \omega; \quad (2)$$

$$D_t \mathcal{C} = \mathcal{C} \cdot (\nabla \mathbf{u}) + (\nabla \mathbf{u})^T \cdot \mathcal{C} - \frac{f(r_P) \mathcal{C} - \mathcal{I}}{\tau_P}. \quad (3)$$

Here, $D_t \equiv \partial_t + \mathbf{u} \cdot \nabla$, the uniform solvent density $\rho = 1$; α is the non-dimensionalised friction coefficient, ν is the kinematic viscosity, and $F_\omega \equiv -n^3 [\cos(nx) + \cos(ny)] / \Omega$, is the non-dimensionalised force with injection wave vector n , $\Omega = n F_{amp} / (\nu^2 k^3)$, and $\alpha = n \nu \alpha' k / F_{amp}$, where F_{amp} is the forcing amplitude, α' is the friction coefficient, and lengths are non-dimensionalised via a factor k/n , with k a wave number or inverse length [4]; $(\nabla \mathbf{u})^T$ is the transpose of $(\nabla \mathbf{u})$, $\mathcal{C}_{\alpha\beta} \equiv \langle R_\alpha R_\beta \rangle$ the elements of the polymer-conformation tensor \mathcal{C} (angular brackets indicate an average over polymer configurations), \mathcal{I} the identity tensor with elements $\delta_{\alpha\beta}$, $f(r_P) \equiv (L^2 - 2)/(L^2 - r_P^2)$ the FENE-P potential that ensures finite extensibility, $r_P \equiv \sqrt{\text{Tr}(\mathcal{C})}$ and L the length and the maximum possible extension, respectively, of the polymers, and $c \equiv \mu / (\nu + \mu)$ a dimensionless measure of the polymer concentration [5]; $c = 0.1$ corresponds, roughly, to 100 ppm for polyethylene oxide [6]. We define the Taylor-microscale Reynolds number as $Re \equiv \sqrt{2\mathcal{E}} / \sqrt{\nu \epsilon}$ and the Weissenberg number as $Wi \equiv \tau_P \sqrt{\epsilon / \nu}$, where \mathcal{E} is the total kinetic energy of the fluid and ϵ the energy dissipation rate per unit mass for the fluid.

NUMERICAL METHOD

We study homogeneous, isotropic, turbulence, so we use periodic boundary conditions and solve Eqs. (1)-(3) by using N^2 collocation points in a square domain (side $\mathcal{L} = 2\pi$). We use a fourth-order, Runge-Kutta scheme for time marching with a time step $\delta t = 0.005$ and an explicit, fourth-order, central-finite-difference scheme in space and the Kurganov-Tadmor (KT) [7] shock-capturing scheme to calculate the advection term of Eq. (3). We solve Eq. (2) in Fourier space by using the FFTW fast-Fourier-transform library. The numerical error in r_P must be controlled by choosing a small time step δt , to prevent r_P from becoming larger than L . We preserve the symmetric-positive-definite (SPD) nature of \mathcal{C} at all times

by adapting to two dimensions the Cholesky-decomposition scheme of Refs. [3, 5] for three-dimensional fluid turbulence with polymer additives. In most of our studies we use $N = 128$; however, we have checked in representative cases that our results are unchanged if we use $N = 256$. We set $c = 0.2$ in all our simulations. To make contact with earlier linear-stability and DNS studies of this problem without polymers, the results we present below have been obtained with no friction. Our qualitative conclusions are not affected by this. We have varied Ω from 1 to 30 and τ_P from 0.1 to 20 to study laminar, dissipation-reduction, and elastic-turbulence regimes.

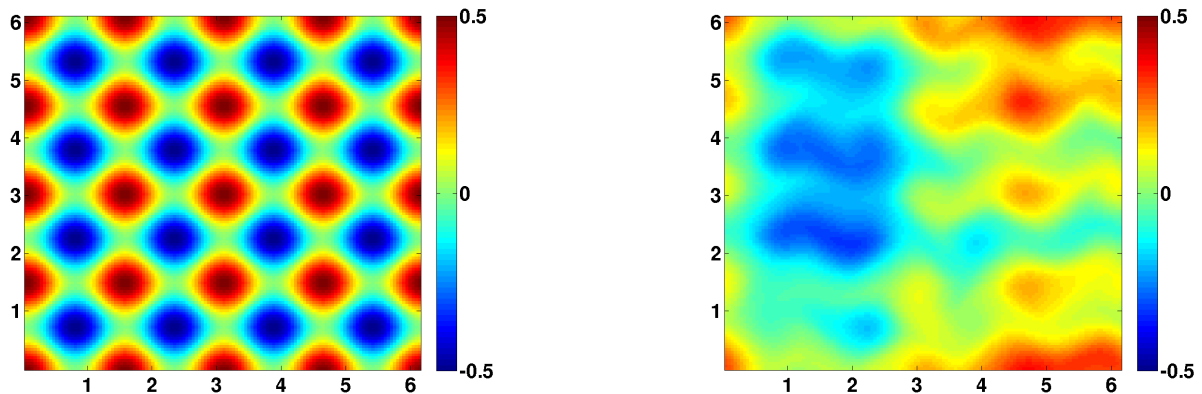


Figure 1. (Color online) Pseudocolour plots of the stream-function field ψ , illustrating the vortex-anti-vortex crystal for $\Omega = 1$: Left panel: no polymers; right panel: with polymers and $\tau_P = 20$.

RESULTS

Our direct numerical simulation (DNS) of Eqs. (1) and (3) yields a variety of interesting results that we summarize below. We find that, as we increase τ_P , with the value of Ω less than critical value at which turbulence occurs without polymers, the vortex-anti-vortex crystal becomes unstable. It goes through a rich sequence of transitions that lead, eventually, to a disordered, elastic-turbulent state. We start from the original, steady, square crystal (SX), imposed by the force; this is followed by steady crystals that are distorted, via large-scale spatial undulations (SXA), relative to SX; these give way to distorted crystals that oscillate in time, either periodically (OPXA) or quasiperiodically (OQPXA); finally, the system becomes disordered and displays spatiotemporal chaos and turbulence (SCT). The pseudocolor plots in Fig. (1) show two extreme cases: the left panel depicts the crystalline structure of ψ when there are no polymer additives and $\Omega = 1$; and the right panel exhibits the melted, elastic-turbulent state, when we added polymers to the fluid, with $\Omega = 1$ and $\tau_P = 20$. As the value of Ω increases, the transition to the SCT state occurs at ever smaller values of τ_P . To obtain the nonequilibrium phase diagram for this system in the $Wi - Re$ plane, we obtain energy time series, various power spectra, and Poincaré-type maps. To characterize spatial correlations in the crystalline and disordered states we obtain the spatial autocorrelation function $G(r) = \langle \overline{\Lambda(\mathbf{x} + \mathbf{r})\Lambda(\mathbf{x})} \rangle$. In the crystalline case, along a given line, this correlation function exhibits a periodic array of peaks; the widths of these peaks are related to the widths of vortical or strain-dominated regions. In the turbulent phase, these peaks decay over a length scale that indicates the degree of short-range order. This decay is similar to the decay of spatial correlation functions in a disordered liquid.

References

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