INFLUENCE OF A STRONGLY SHEAR-THINNING RHEOLOGY ON NONLINEAR WAVES WITH A 3-FOLD ROTATIONAL SYMMETRY IN PIPE FLOW: ASYMPTOTIC REGIME

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Abstract The influence of a shear-thinning rheology on nonlinear waves with a 3-fold rotational symmetry in pipe flow is studied. We focus on the family of waves discovered by Faisst & Eckhardt in 2003, Wedin & Kerswell in 2004. The Carreau model, which is quite regular, is chosen to describe the rheology of the fluid. The pseudo-spectral code of Roland et al. 2010 is used to compute the nonlinear waves, by continuation, starting from the Newtonian case. The retardation effect found in 2010 is studied in a more systematic manner: the influence of the axial wavenumber is analyzed. An asymptotic regime is discovered in the limit of quite strong shear-thinning effects, where the fluid behaves like a power-law fluid. If one admits that the nonlinear waves are 'precursors' of turbulence, this gives a lower bound for the onset of turbulence in the pipe flow of some Carreau and power-law fluids.

INTRODUCTION

The transition to turbulence in pipe flow is difficult to model, even in Newtonian fluids, because of its highly nonlinear nature. A new path has been opened recently by [3, 6] to attack this problem. It consists in computing 'exact coherent structures' which are nonlinear traveling waves. For each family of waves, there is a critical Reynolds number below which no such waves exist, and at which a first wave emerges through a saddle-node bifurcation. We focus here on the waves found by [3, 6] with a 3-fold rotational symmetry, i.e., if (r, θ, z) are the cylindrical coordinates with z the axis of revolution of the pipe, the waves are invariant under $\theta \mapsto \theta + 2\pi/3$. The waves are also invariant under $z \mapsto z + 2\pi/q$ with q the axial wavenumber. For these waves the critical Reynolds number, based on the mean flow velocity $\overline{W}$, the pipe radius a and the kinematic viscosity ν, is

$$Re \;=\; 2a\overline{W}/\nu \;=\; 1251\;. \tag{1}$$

This is a lower bound of the Reynolds numbers at which turbulence exists. Moreover, some experiments have shown that, in 'puffs', the flow structure can transiently approach the one of the nonlinear traveling waves computed numerically [4]. For all these reasons, these nonlinear waves may be viewed as 'precursors' of turbulence.

In non-Newtonian fluids, a delay for the onset of developed turbulence in pipes has been evidenced experimentally by several authors, e.g. [1, 2]. Most non-Newtonian fluids are shear-thinning and viscoelastic. Here we focus on the influence of the shear-thinning effects, neglecting the elastic response of the fluid, which has been the effect of a lot of attention in the literature. By computing nonlinear waves of the family of the ones found by [3, 6], we obtain a model of the transition delay found experimentally.

MODEL AND METHODS

Most of the non-Newtonian fluids used experimentally exhibit strong shear-thinning effects, for which the power-law or Cross or Carreau-Yasuda rheological models are relevant. In all these models, the viscosity at zero rate-of-strain is not defined (power-law) or not differentiable (Cross or Carreau-Yasuda). In order to have, from a mathematical point of view, a well-posed problem, we consider instead a Carreau model, for which the viscosity dependence on the velocity field is C^∞:

$$\nu \;=\; \nu_0\,(1 \;+\; \lambda^2 D_2)^{(n-1)/2} \tag{2}$$

with ν_0 the viscosity at rest, λ the characteristic time of the fluid, $n < 1$ the shear-thinning index, D_2 the second invariant of the rate-of-strain tensor. In our computations, $n = 1/2$. A relevant time unit is the advection time $t_a = a/W_0$ with W_0 the centerline velocity of the laminar flow at the mean pressure gradient that is applied. When $\lambda = 0$, a Newtonian fluid is recovered. When $\lambda \gg t_a^{-1}$, the laminar flow approaches the one of a power-law fluid

$$\nu \;=\; \nu_0\,\lambda^{n-1}\,D_2^{(n-1)/2}\;, \tag{3}$$

i.e., the fluid behaviour approaches the behaviour of a power-law fluid. Experimentally, the relevant Reynolds number is the one based on the wall-viscosity ν_w,

$$Re_w \;=\; 2a\overline{W}/\nu_w\;. \tag{4}$$

This viscosity can be determined from a measurement of the wall pressure gradient, which gives access to the wall shear stress. The rheological law then gives ν_w. The transition delay advocated in our Introduction holds when Re_w numbers are used. From a theoretical point of view, as soon as $\lambda \gtrsim 2t_a$, the power-law fluid formula

$$\nu_w \;=\; \nu_0\,\lambda^{n-1}\,(1 + 1/n)^{n-1} \tag{5}$$

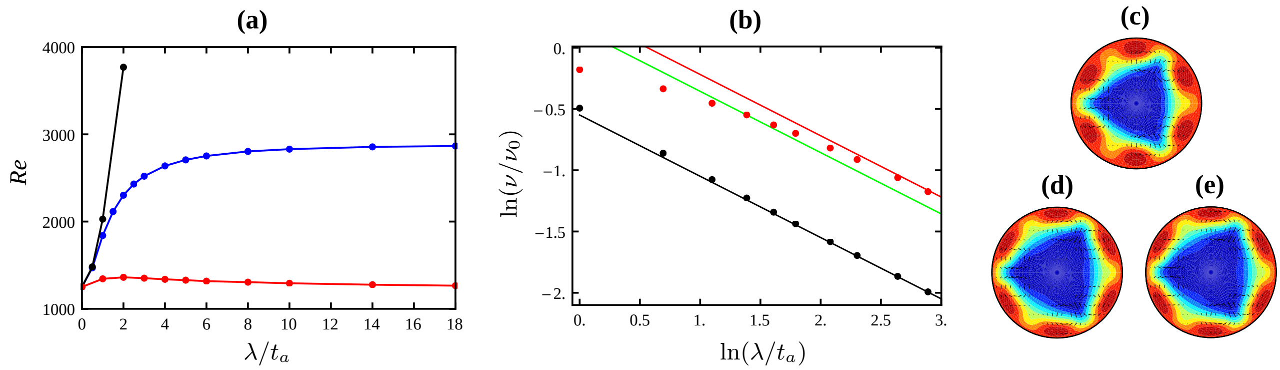

Figure 1. (a) : Critical Reynolds numbers for the onset of the waves computed at $q = q_N$ or at optimal q. Black: Re_w for $q = q_N$; blue: Re_w for optimal q; red: Re_m for optimal q. **(b) :** Logarithm of the wall viscosity ν_w numerically computed (black disks) or calculated from the formula (5) (black line); logarithm of the mean viscosity ν_m numerically computed (red disks) and fitted (6) (red line); logarithm of the mean viscosity ν_{mb} in the corresponding power-law fluid laminar base flow (green line). **(c, d, e) :** Velocity fields averaged over z for the critical waves at optimal q for $\lambda = 0$ **(c)**, $4t_a$ **(d)**, $8t_a$ **(e)**. The colors show the difference between the mean axial velocity of the waves and the corresponding laminar flow; the arrows show the mean flow in the section.

gives a good estimate of ν_w [Black data Fig. 1b]. A pseudo spectral code has been developed to compute nonlinear waves in the pipe flow of Carreau fluids [5]. The first results obtained, for which the axial wavenumber q was set at its critical value for Newtonian fluids, $q = q_N = 2.44/a$, showed a quite strong retardation effect [Black data Fig. 1a]. Because of the large Reynolds numbers attained at $\lambda = 2t_a$, which require a high resolution, larger values of λ were not studied.

RESULTS

By varying the axial wavenumber q, in order to minimize Re_w, nonlinear waves are found, which occur at quite smaller values of Re_w, as compared with the ones computed for $q = q_N$ [Blue data Fig. 1a]. When $\lambda \to +\infty$, Re_w and the velocity field of the critical waves converge towards an asymptotic limit [Fig. 1d,e]. Accordingly the critical wavenumber tends to $q_\infty = 1.99/a$, the phase velocity to $0.485W_0$ and the mean velocity to $0.415W_0$. A computation of the viscosity in the nonlinear critical wave flows, averaged in the volume of the pipe, i.e. the mean viscosity ν_m, demonstrates [Red data Fig. 1b] the existence of an asymptotic law of a form similar to (5): as $\lambda \to +\infty$,

$$\nu_m \sim \nu_1 \lambda^{n-1} . \tag{6}$$

The mean viscosity ν_m is the relevant viscosity for these waves. Indeed, the Reynolds number based on this viscosity,

$$Re_m = 2a\overline{W}/\nu_m , \tag{7}$$

stays almost constant whatever λ, from $\lambda = 0$ (Newtonian fluid) to $\lambda \to +\infty$ (power-law fluid) [Red data Fig. 1a]:

$$Re_m = 2a\overline{W}/\nu_m \simeq Re(\lambda = 0) = 1251 . \tag{8}$$

This surprisingly simple result, and the fact that the mean viscosity in the waves is of the order of the mean viscosity in the base laminar flow of the corresponding power-law fluid, $\nu_{mb} = \nu_2 \lambda^{n-1}$, which can be computed analytically [Green data Fig. 1b], yields an approximate analytic formula for the critical Reynolds number of the waves in the asymptotic regime,

$$Re_w = (\nu_m/\nu_w) Re_m \simeq 1251 (\nu_{mb}/\nu_w) . \tag{9}$$

This formula may be used for Carreau and power-law fluids with $n > 1/3$, for ν_{mb} to be defined.

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