STOCHASTIC DECOMPOSITION OF ATMOSPHERIC TURBULENCE

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Abstract Gustiness and pronounced intermittent behavior of wind speed measurements in the turbulent atmospheric boundary layer are analysed. A new set of parameters is worked out which finally shows that the two-time statistics of atmospheric turbulence is closely linked to the properties of homogeneous isotropic turbulence in the sense of Kolmogorov 62.

INTRODUCTION

The atmosphere is a system of high complexity, subjected to a manifold of influences, such as gravity, surface friction, heating and cooling. Consequently, atmospheric turbulence presents itself as a very complex phenomenon of fluid dynamics. Moreover, atmospheric turbulence is of high relevance for a number of technical applications. One example is the conversion of kinetic wind energy into electrical power, which necessarily takes place in the turbulent atmospheric boundary layer. Therefore wind energy converters are permanently exposed to atmospheric turbulence.

One of the striking features of atmospheric turbulence is the pronounced intermittency of velocity increments $u_r = u(t+\tau) - u(t)$, in the sense of very high probabilities for extreme increment values, compared to a Gaussian assumption. For homogeneous isotropic turbulence (HIT) in laboratory experiments this intermittency is pronounced only on small scales and vanishes completely at the integral scale. In atmospheric turbulence (AT), in contrary, strong intermittency is observed even for very large scales such as several hours, corresponding to several 10 kilometers of spatial separation. Intermittency, defined as above, corresponds to high probabilities of extreme changes in wind velocity. This can be regarded as a mathematical approach to what is commonly called gusts. We investigate the question how this gustiness is related to the known intermittency of homogeneous isotropic turbulence.

To this end we develop a superposition approach inspired by the works of Castaing e.a. [4], Boettcher e.a. [3], and Beck [2]. The basic idea of our ansatz is to work out important parameters which allow to decompose AT into elements of HIT.

DECOMPOSITION APPROACH

A superposition model for increment PDFs of HIT was originally introduced by Castaing e.a. [4], who considered the intermittent increment PDFs as a sum of Gaussian PDFs with different standard deviations. The standard deviations of these Gaussians themselves were assumed to be log-normally distributed, leading to a straightforward integral expression for the increment PDFs:

$$p(u_\tau) = \frac{1}{2\pi\lambda(\tau)} \int_0^\infty \frac{\mathrm{d}\sigma}{\sigma^2} \exp\left[-\frac{u_\tau^2}{2\sigma^2}\right] \exp\left[-\frac{\log^2(\sigma/\sigma_0)}{2\lambda^2(\tau)}\right] . \tag{1}$$

Two parameters characterize this expression: first, the intermittency parameter λ^2, which defines the deviation of the resulting PDF from a Gaussian, and second, the most probable standard deviation σ_0 within the integral. According to Kolmogorov's refined scaling hypothesis (K62) [5] an intermittency factor μ governs the nonlinear scaling behavior of the structure functions

$$S^n(\tau) = \langle [u(t+\tau) - u(t)]^n \rangle \sim \tau^{\zeta_n} \quad \text{with} \tag{2}$$

$$\zeta_n = \frac{n}{3} + \frac{\mu}{18}(3n - n^2) , \tag{3}$$

where $\mu = 0.26 \pm 0.04$ [1]. A relation between λ^2 and μ, up to a constant Λ_0, can in the context of K62 straightforwardly be obtained as

$$\lambda^2(\tau) = \Lambda_0 - \mu \log(\tau) . \tag{4}$$

Later, Beck [2] showed that for the case of a strictly log-normal distribution of standard deviations, both parameters can be derived rigorously from the flatness of the time series. Boettcher e.a. [3] applied these findings to atmospheric flows, introducing an additional condition on the ten-minute mean wind speed $\bar{u} = \langle u(t) \rangle_{10\,\mathrm{min}}$.

In this contribution we suggest a new normalization of the velocity increments on the standard deviation of the so-called fluctuations $u' = u(t) - \bar{u}$, leading to $\tilde{u}_\tau = u_\tau / \sigma_{u'}$. This $\sigma_{u'}$ can be understood as a simplified measure for the turbulent kinetic energy (TKE). In analogy to [4, 3] and (1) we obtain the expression

$$p(\tilde{u}_\tau|\bar{u}) = \int \mathrm{d}(\log \sigma_{u'})\, LN(\sigma_{u'}|\bar{u})\, p(\tilde{u}_\tau|\bar{u}, \sigma_{u'}) , \tag{5}$$

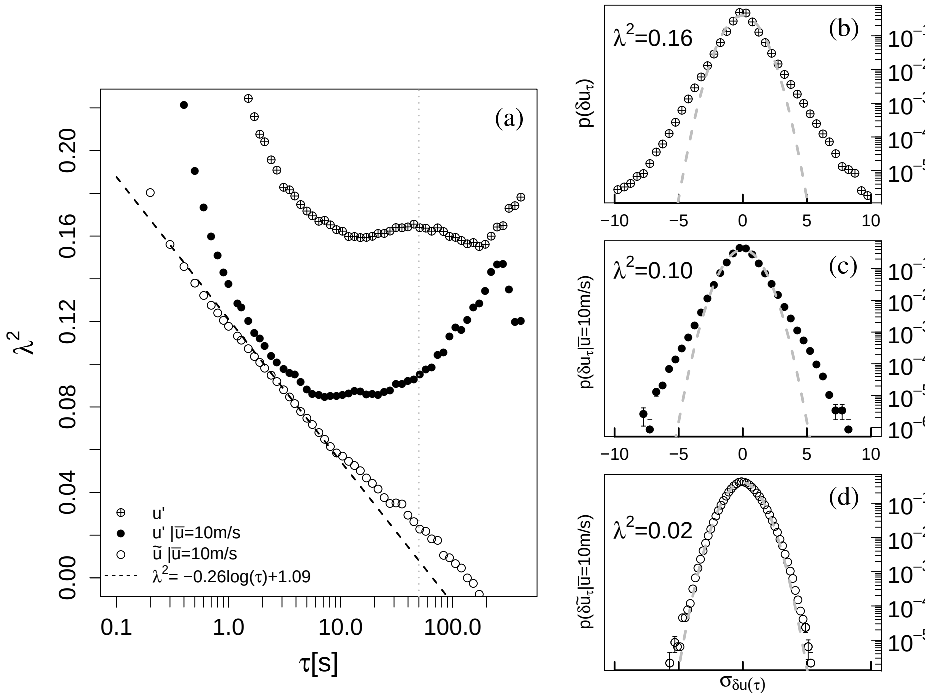

Figure 1. Statistcal properties of velocity increments for different conditioning. (a) Dependency of the intermittency parameter $\lambda^2(\tau)$. A dashed line indicates a logarithmic law $\lambda^2 \sim 0.26\log(\tau)$ according to K62. (b) to (d) Probability densities at $\tau = 50\,\mathrm{s}$.

were $LN(\cdot)$ is an abbreviation for the log-normal distribution and $p(\tilde{u}_\tau|\bar{u},\sigma_{u'})$ is of the form of Eq. (1).

In Fig. 1(a) the dependence of $\lambda^2(\tau)$ for wind velocity increment PDFs is shown for a data set of one month from the German FINO 1 offshore platform[1], measured by a cup anemometer at the mast top in a height of about 100 m. Three examples of different conditioning are presented: the topmost curve is derived directly from unfiltered increment PDFs, for the middle one a condition to $\bar{u} = 10\,$m/s was applied, according to [3]. For the lower curve the velocity increments were additionally conditioned on $\sigma_{u'}$, following our new approach as described above. The dashed line indicates behavior according to K62 with $\lambda^2(\tau) = 1.09 - 0.26\log(\tau)$.

CONCLUSIONS

While for the unfiltered increment PDFs as well as for the conditioning on the mean wind speed the dependence of $\lambda^2(\tau)$ is far from the K62 prediction for HIT, the new approach with additional normalization by $\sigma_{u'}$ obtains results very close to the known HIT scaling $\lambda^2(\tau) \sim 0.26\log(\tau)$. We conclude that by introducing this normalization we can actually decompose AT into components which have properties of HIT, and which are defined by the term $p(u_\tau|\bar{u},\sigma_{u'})$ in (5). Coming back to the question of the origin of gusts, we find that they are stemming directly from fundamental properties of small scale turbulence and there is no indication of qualitatively new coherent stuctures.

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[1] cf. http://www.fino1.de/en/