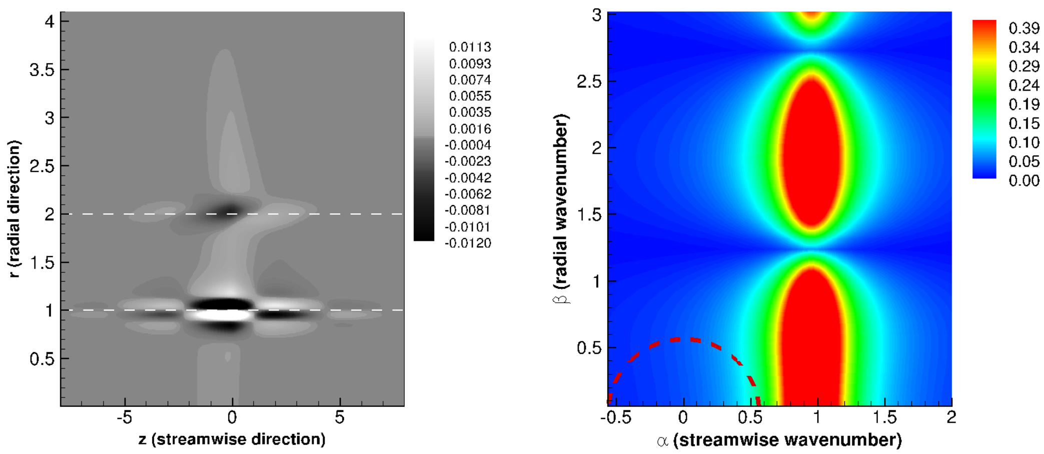
NOISE RADIATION FROM INSTABILITY WAVES IN SUBSONIC COAXIAL JETS

Gloor Michael, Obrist Dominik & Kleiser Leonhard
Institute of Fluid Dynamics, ETH Zurich, Zurich, Switzerland

Abstract Classical aeroacoustic theory for free shear flows often relies on quadrupole point-sources to represent the sound sources originating from an unsteady flow field. However, because of almost complete mutual cancellation of the sources [1], such a point-wise analysis of the flow field is usually neither practical for numerical analysis nor does it provide good insight into the noise generation mechanisms induced by large-scale instability waves which are responsible for the dominant downstream noise radiation in subsonic jets. We model such instability waves in coaxial jets using linear stability theory and show that frequency-specific sound radiation patterns can be explained by this simplified approach.

INTRODUCTION

It is well known that the intensity and frequency composition of the sound field emitted from subsonic transitional and turbulent jets exhibits a strong dependency on the observation angle [2]. Several studies have recently identified wavepacket-like structures within the unsteady flow field as the dominant source responsible for sound radiation at low angles with respect to the downstream direction [3, 4, 5]. These wavepacket sources can be associated with subsonic instability waves that produce sound due to their streamwise amplitude variation. While constant-amplitude waves with subsonic phase velocities would not lead to sound radiation, the streamwise growth, saturation and decay leads to a so-called "supersonic tail" in the axial wavenumber spectrum for jets at moderate to high subsonic Mach numbers [6]. It is easy to show that the disturbance energy contained in the supersonic wavenumber range leads to directive sound radiation. The spectral formulation of Lighthill's acoustic analogy [7] provides an elegant way to predict the sound radiation from wavepacket sources by using geometrical considerations in the wavenumber-frequency space.



(a) (b)

Figure 1. (a) Lighthill source $q(z, r)$ for an axisymmetric wavepacket disturbance in a coaxial jet flow ($St_D = 0.2$). The dashed horizontal lines indicate the radial location of the velocity shear layers; (b) Lighthill source $\hat{q}(\alpha, \beta)$ in wavenumber space.

METHODS

Lighthill's acoustic analogy [8] shows that the Navier-Stokes equations can be recast into a wave operator acting on the density field ρ and a source term q:

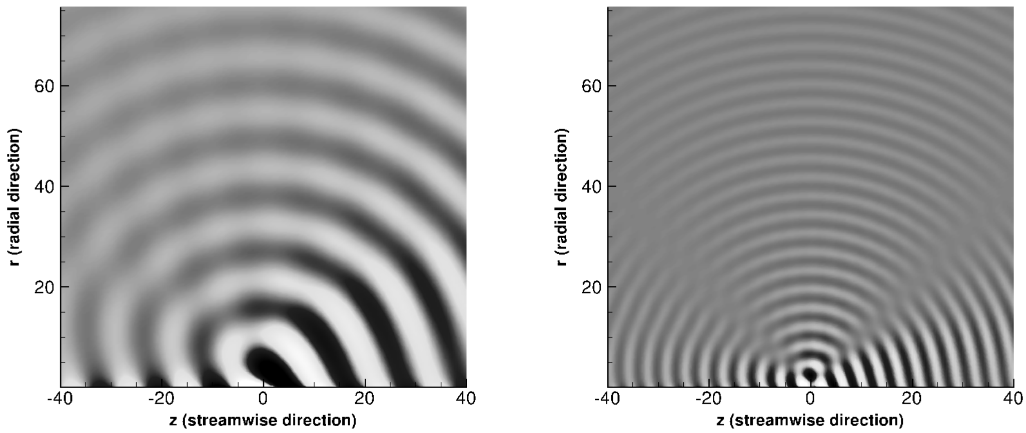
$$\frac{\partial^2 \rho}{\partial t^2} - a_\infty^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = q, \tag{1}$$

where a_∞ is the ambient speed of sound, $q = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$ and T_{ij} is the Lighthill stress tensor. The main objective of this study is the modeling of instability waves which can be observed in transitional and turbulent jets and to predict the directivity of the emitted sound. To this end we use a simplified model for the representation of the source term q of Eq. (1) by representing the radial structure of the wavepacket disturbances by instability modes obtained from a local spatial linear stability analysis and by prescribing an analytical axial wavepacket envelope based on a study by Crighton & Huerre [9].

The source q will take the form of a wavepacket in physical space as well as in a spectral representation. Depending on the spectral shape of the wavepacket and its relative location (in wavenumber space) with respect to the acoustic dispersion relation, the directivity pattern of the radiated sound can be predicted for different instability modes [7]. Comparisons with data obtained from Direct Numerical Simulations are performed to justify our modeling approach.

RESULTS

We numerically solve the spatial linear stability problem for a compressible flow at high subsonic Mach numbers to obtain the frequency-dependent instability modes. For the investigated coaxial jet flow, two types of unstable modes can be identified. The first mode can be associated with the inner shear layer while the second mode is responsible for disturbance development in the outer shear layer. These subsonic modes, which can be associated with Kelvin-Helmholtz instabilities, only radiate due to their axial amplitude modulation. To investigate the acoustic radiation from these subsonic instability waves, axial wavepacket envelopes are prescribed to model the streamwise evolution of the instabilities.
Fig. 1(a) presents the Lighthill source q computed for the Strouhal number $St_D = 0.2$ whereas Fig. 1(b) shows the same source in wavenumber space. The acoustic far-field dispersion relation is indicated by a dashed line ("the acoustic circle"). The instability modes are always centered to the right of the acoustic circle in wavenumber space due to the subsonic phase velocity of the carrier wave. The axial envelope defines the extent in α-direction whereas the radial structure of the source q defines the spectral shape of the wavepacket in β-direction. Because of the axial amplitude modulation of the instability wave, an overlap of the disturbance wavepacket with the acoustic circle occurs in the wavenumber-frequency space. The intensity of the overlap along the acoustic circle ultimately defines the directivity of the noise radiation. The noise radiation pattern in physical space can then be computed using the spectral formulation of Lighthill's analogy. Results for instability modes at different Strouhal numbers are illustrated in Fig. 2. The clear downstream dominance of the sound radiation for low-frequency modes, an effect which was also observed in simulations and experiments [3], is illustrated in Fig. 2(a) for $St_D = 0.2$. The sound radiation for higher frequencies is less directive (c.f. Fig. 2(b) for $St_D = 0.6$).



(a) $St_D = 0.2$

(b) $St_D = 0.6$

Figure 2. Acoustic pressure fluctuations due to wavepacket disturbances in coaxial jet flow for two different Strouhal numbers: (a): $St_D = 0.2$ and (b): $St_D = 0.6$. The sound field is computed by the spectral formulation of Lighthill's acoustic analogy [7].

References

[1] J. B. Freund. Noise-source turbulence statistics and the noise from a Mach 0.9 jet. *Phys. Fluids*, **15**(6):1788, 2003.
[2] K. Viswanathan. Mechanisms of jet noise generation: classical theories and recent developments. *Int. J. Aeroacoutics*, **8**(4):355–407, 2009.
[3] P. Jordan and T. Colonius. Wave Packets and Turbulent Jet Noise. *Annu. Rev. Fluid Mech.*, **45**(1):120920150849001, 2012.
[4] A. V. G. Cavalieri, P. Jordan, T. Colonius, and Y. Gervais. Axisymmetric superdirectivity in subsonic jets. *J. Fluid Mech.*, **704**:388–420, 2012.
[5] J. B. Freund. Noise sources in a low-Reynolds-number turbulent jet at Mach 0.9. *J. Fluid Mech.*, **438**:277–305, 2001.
[6] V. Suponitsky, N. D. Sandham, and C. L. Morfey. Linear and nonlinear mechanisms of sound radiation by instability waves in subsonic jets. *J. Fluid Mech.*, **658**:509–538, 2010.
[7] D. Obrist. Directivity of acoustic emissions from wave packets to the far field. *J. Fluid Mech.*, **640**:165, 2009.
[8] M. J. Lighthill. On Sound Generated Aerodynamically. I. General Theory. *Proc. R. Soc. A*, **211**(1107):564–587, 1952.
[9] D. G. Crighton and P. Huerre. Shear-layer pressure fluctuations and superdirective acoustic sources. *J. Fluid Mech.*, **220**(1):355–368, 1990.