# INERTIAL WAVES AND WAVE EXCITATION MECHANISMS IN ANNULAR CAVITIES: SIMULATIONS, EXPERIMENTS AND THEORY

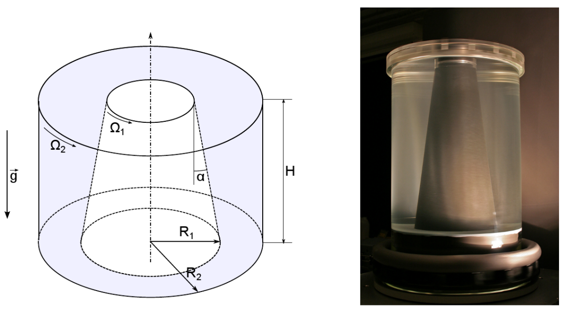
M. Klein[1], I. D. Borcia[2], C. Egbers[2], A. Ghasemi V.[1], U. Harlander[2], M. V. Kurgansky[3], E. Schaller[1], T. Seelig[2], and A. Will[1]

[1]*Chair of Environmental Meteorology, Brandenburg University of Technology, Cottbus, Germany*
[2]*Chair of Aerodynamics and Fluid Mechanics, Brandenburg University of Technology, Cottbus, Germany*
[3]*A. M. Obukhov Institute of Atmospheric Physics, Russian Academy of Sciences, Moscow, Russia*

Recently, wave excitation mechanisms, wave attractors and the linkage to instabilities have been discussed for spherical shells, cylinders and cubes, e. g. [1], [2], [3], [4]. Especially spherical shells have been heavily investigated, since they are of immediate importance for planetary flows. Also cylinders and annuli have been studied by many researchers due to the simpler geometry. However, spherical shells exhibit a large number of singularities in their mode spectrum in contrast to straight-walled cylinders and annuli. We therefore investigated annular geometries, whose perfect rectangular symmetry in the cross-sectional radial plane has been broken in order to recover the situation of a spherical shell in a controlled manner. To our knowledge, a comprehensive study of wave excitation mechanisms in such annular geometries has not yet been conducted. In order to contribute to the ongoing discussion of wave excitation mechanisms, we bring together numerical simulations, laboratory experiments and theoretical analysis.

The set-up we proposed is quite similar to a classical Taylor-Couette system, i. e. the annular tank is filled with a homogeneous fluid and the flow domain is bounded by two co-axial cylinders and two lids. This classical set-up requires a subtle change in order to allow (i) the existence of wave attractor solutions and (ii) the investigation of the effects of a librating wall that intersects the axis of rotation at an angle of $\alpha = 5.7°$ (cf. fig. 1). As shown in fig. 1, we accomplish this by using a conical frustum in place of an inner cylinder. Due to the peculiar reflection characteristics of inertial waves, in which the angle to the rotational axis is preserved (see e. g. [1]), we expect wave attractor solutions similar to those predicted in [5]. The whole annulus rotates with a mean rotation rate $\Omega_0$, so that inertial waves with frequencies $0 < \omega < 2\Omega_0$ can be excited. Waves of predominant frequency $\omega$ are generated by longitudinal libration of at least one of the four bounding walls. For the librating parts an additional time-harmonic term is added, so that their rotation rate becomes $\Omega(t) = \Omega_0\big[1 + \varepsilon\sin(\omega t)\big]$, where $\varepsilon$ is the libration amplitude. In the laboratory set-up the lids are attached to the outer cylinder, which means two separated parts can rotate independently with $\Omega_1$ and $\Omega_2$ respectively. With an inclined inner cylinder we have opened the possibility to investigate wave excitation efficiency in dependence on the inclination angle of the librating surface by having a constant Coriolis parameter on the sloping surface. Our analytical investigations suggest that there is an Ekman layer on the sloping wall with a reduced Coriolis parameter $0 < f' < 2\Omega_0$ with $f' = 2\Omega_0 \sin\alpha$. Consequently we expect a strong frequency dependency in the inertial wave band. This investigation seems to be impossible in rectangular geometries, because only Ekman layers on the lids will be present. On the other hand, it is at least very difficult to perform the investigation in spherical configurations, since all possible angles are present. This leads to the so-called boundary layer eruption [6] for any frequency in the inertial band, so that only localized wave excitation at the critical angle $\sin\theta_c = \omega/2\Omega_0$ can be observed (see e. g. [1]).

In order to resolve the different wave excitation processes within a numerical model, fully three-dimensional direct numerical simulation (3D-DNS) of the common set-up have been performed. To this end, we used a code that solves the incompressible Navier-Stokes equations for the volume flux components in generalized curvilinear coordinates. The numerical scheme conserves mass, momentum and kinetic energy if the grid coordinates fulfill local orthogonality. In




(a) (b)

**Figure 1.** Sketch (a) and photograph (b) of the standard set-up with a conical frustum in upright position. The inner and outer wall can rotate with different rotation rates $\Omega_1$, $\Omega_2$.

addition to the standard set-up (fig. 1) we have also investigated configurations that are not accessible in the laboratory. The separation between possible wave excitation mechanisms involved the simulation of axially periodic domains and cases with different sets of librating walls.

The results we obtained so far exhibit qualitative agreement between simulations, theory and laboratory experiments, whereas also quantitative agreement was observed between theory and numerical results (fig. 2, (a–c)). We believe that the differences in measurements can be attributed to the observation of other variables than pressure and velocity, and optical errors, like lens effects, distort the patterns recorded. For the results shown in fig. 2 (b) an optical flow field visualization technique with immersed particles was used. Particle image velocimetry (PIV) measurements are planned in order to obtain information about the velocity field, which should open the possibility of quantitative evaluations of simulations and theory. Nevertheless, for $\varepsilon \approx 10^{-1}$ and Ekman numbers as small as $E = \nu/\Omega_0\overline{\Delta r} \approx 10^{-5}$, for instance, we were able to observe wave attractor geometries predicted in [5] in the numerical simulations and measurements. Up until now, theory, numerics and laboratory experiments were found to exhibit the same qualitative frequency dependency of excited waves in case of the librating conical frustum (fig. 2, (d)).

In the talk the results from numerical simulation and measurements will be presented for the configurations that have been investigated. The results will furthermore be compared with analytical solutions, which were obtained by the methods of characteristics and from the boundary layer equations. While emphasis will be placed on wave excitation and wave attractors, certain aspects of instabilities, e. g. the onset of a transitional regime, will be touched as well.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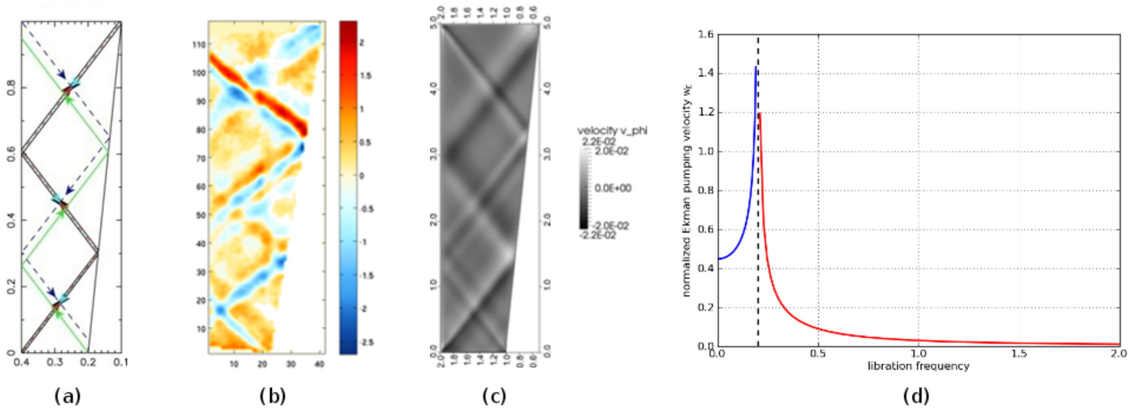


**Figure 2.** Wave beam excited in the standard set-up via libration of the outer cylinder and lids. Fig. (a) shows the theoretical prediction of wave beam trajectories for the first two reflections coming out of the corners (colored and dashed lines) and the resulting limit cycle (solid black line) [5]. In (b) a temporal snapshot of the reconstructed time series is shown. Raw data has been fitted with a Fourier series that contains frequencies $\omega_n = n\omega < 2\Omega_0$ ($n = 0, 1, 2, \ldots$). We filter the excitation frequency by using only $n = 1$ components for the reconstruction. Fig. (c) shows a corresponding temporal snapshot of the azimuthal velocity field $v_\phi$ from a numerical simulation with similar parameters. Parameters for the laboratory and numerical experiments are Ekman number $E \approx 2 \cdot 10^{-5}$, libration frequency $\omega/\Omega_0 = 1.21$, libration amplitude $\varepsilon_{\mathrm{exp}} = 0.2$ in (b) and $\varepsilon_{\mathrm{num}} = 0.1$ in (c) respectively. In (d) the analytically expected frequency dependence of the excitation efficiency for a librating inner wall is plotted.

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