PATTERNED TURBULENCE AND RELAMINARIZATION IN MHD PIPE AND DUCT FLOWS

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Abstract We present results of numerical analysis of relaminarization processes in MHD ducts and pipes flows at moderate values of the Reynolds and Hartmann numbers. The analysis also pursues a historical purpose of reproducing Julius Hartmann's experiments on flows of mercury in pipes and ducts under the influence of magnetic fields. The computed critical parameters for the laminar-turbulent transition as well as the friction coefficients are in excellent agreement with Hartmann's data. The simulations provide a first detailed view of flow structures that are experimentally inaccessible. Novel flow regimes with localized turbulent spots near the side walls parallel to the magnetic field are discovered.

INTRODUCTION

The processes of flow re-laminarization in tubes (i.e. pipes and ducts) were the first MHD phenomena studied experimentally by the work of Hartmann & Lazarus [1]. The experimental settings were pipes and ducts of different aspect ratios, subjected to a uniform transverse magnetic field. The flow regimes were represented by moderate values of the Reynolds number and magnetic fields $\boldsymbol{B}$ corresponding to $Ha = 0 \ldots 30$. Complete flow laminarization was detected for the range $R \approx 220$, where $R \equiv Re/Ha$ is the Reynolds number based on the Hartmann layer thickness δ_{Ha}. For the laminar MHD pipe and duct flow, the essential feature is a flat core and thin boundary layers: the Hartmann layers of thickness scaling as $\sim Ha^{-1}$ at the walls perpendicular to the magnetic field and the side-wall layers of thickness $\sim Ha^{-1/2}$ (Shercliff layers in duct) or $\sim Ha^{-2/3}$ (Roberts layers in pipe) at the walls parallel to the magnetic field. Both Hartmann and side-wall layers (Shercliff and Roberts) develop strong shear and velocity gradients, and can be viewed as candidates for the flow instability and transition. In the prior study ([2]), based on the analysis of optimal perturbations, we have identified the side-wall layers as the most prominent source of instability. The optimal modes, which provide maximum transient amplification and play a key role in the process of transition, are largely localized at the side walls. In the present study we reproduce numerically the Hartmann & Lazarus experiments and perform a series of relaminarization simulations using fully non-linear DNS.

MATHEMATICAL MODEL AND NUMERICAL METHOD

Flows of incompressible, Newtonian, electrically conducting fluids (e.g. liquid metals) are considered. Based on the assumption of small magnetic Reynolds number Re_m, the flows are described by the quasi-static approximation of MHD equations [3]. The non-dimensional parameters are the Reynolds $Re \equiv Ua/\nu$ and Hartmann $Ha \equiv Ba\,(\sigma/\rho\nu)^{1/2}$ numbers. Here U is the mean flux velocity, a is the half-diameter (pipe) or half-height (duct), σ is the electrical conductivity. The governing equations are solved numerically by our DNS solvers, implemented for rectangular and cylinder geometries. The solvers are based on finite-difference method with collocated grid arrangement, as described in [4]. The spatial discretization of 2^{nd} order is on a non-uniform structured grid formed along the lines of the Cartesian (duct) or cylindrical (pipe) coordinate system. The time integration is explicit and uses standard projection procedure to satisfy incompressibility.

RESULTS AND DISCUSSION

The simulations have been conducted for two conceptually different settings: flows periodic in the streamwise direction and flows with fully non-periodic inlet/exit conditions. Problem setting with periodic conditons can be viewed as the one corresponding to a fully developed flow under perfectly uniform magnetic field. The non-periodic formulation, on the other hand, is more realistic and allows us to apply non-uniform magnetic fields with sharp gradients at the entry and exit of the test sections. By that it is possible to mimic the real flow conditions in experiments, where the magnetic field is never perfectly uniform and the flow evolution is largely influenced by entry effects.

The specific focus of our study, apart from reproducing the classical MHD experiments [1], is on the appearence of patterned turbulence in MHD tubes. The phenomenon of patterned turbulence, i.e. coexistence of laminar and turbulent zones, is known for hydrodynamic wall-bounded shear flows, e.g. puffs and slugs in pipe flow [5, 6] and spiral bands in Taylor-Couette flow [7]. However, coexistence of stable laminar and turbulent regions has not been directly demonstrated for MHD tube flows.

We start with the results of periodic DNS [8], where flows in a pipe and a duct of square cross-section are analyzed at moderate (3000 to 5000) values of *Re*. The key feature of these simulations is the large length of the computational domain, up to 64π in terms of the hydraulic radius a. As a result of that, previously unknown patterned turbulence regimes have been observed for both pipe and duct. The regimes are realized in all DNS conducted within a certain range of *Ha* (e.g., at $Re = 5000$, the range was $21 < Ha < 26$ for duct and $18 < Ha < 23$ for pipe). This range of *Ha* is found to be the transitional one. Below and above it, all the simulations yield fully turbulent or fully laminar flows.

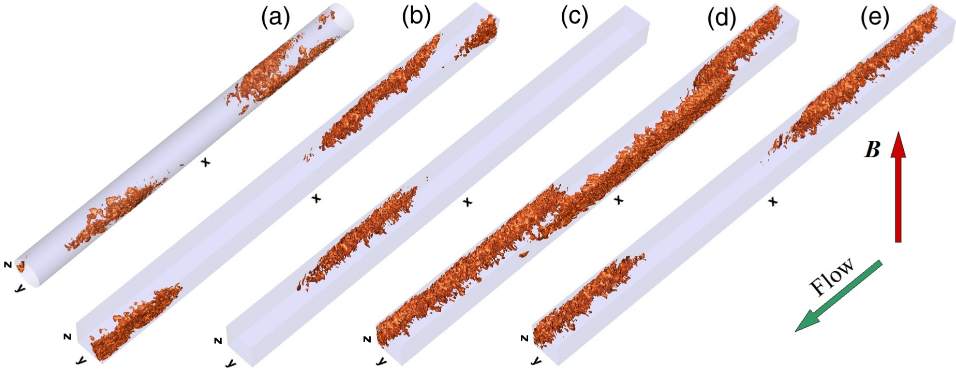

Figure 1. Patterned turbulence regimes in pipe and duct flows in periodic domains. Different flow states at $Re = 5000$ are visualized by iso-surfaces of turbulent kinetic energy of transverse velocity components: puffs in pipe at $Ha = 22$ (a), double- and single-sided puffs in duct at $Ha = 25$ (b, c), and extended turbulent zones induct at $Ha = 22$, cases of double- and single-sided patterns (d, e). The total length of the computational domain is 80 pipe radii in (a) and 32π of duct half-widths in (b)-(e).

The patterned turbulence regimes are illustrated in figure 1. A new feature having no analogues in hydrodynamic systems is the localization of turbulent spots near the sidewalls. The flow in the core and Hartmann boundary layers remains essentially laminar. The feature can be related to the nature of the nonlinear transition mechanism in MHD duct and pipe flows, where, as we have shown by the analysis of optimal perturbations, strong transient growth first occurs in the sidewall layers. The puffs localized are the sidewalls tend to form staggered patterns (1b,d) although the specific arrangement is largely influenced by initial conditions. We have identified multiple events as, e.g., merging and splitting of two or more neighboring puffs, two opposite-side spots forming a 'locked' state and traveling together (1a). In most cases one can identify a characteristic length of a single spot, which is about 30 radii.

The same behavior and pattern formations have been observed in the non-periodic simulations [9]. The only significant difference to the periodic flows is that the arrangement of patterns is less regular, which can be attributed to the inherent features of spatial flow evolution. The formation of puff-patterns shows irregularity and some sort of spatio-temporal intermittency, particularly well seen close to laminarization. We have observed several irregularities, such as events of switch-over of puffs from one side to another, coexistence of staggered and locked patterns, single sporadic spots, as well as events of rapid change from laminar flow to densely populated states. The issues need further analysis and this part of the study is still an ongoing effort.

We have presented new results demonstrating that patterned turbulence is a common feature of MHD tube flows too. Secondly, we have also confirmed that, for the considered range of parameters, relaminarization in MHD tubes occurs around $R = Re/Ha = 200$ ... 220 that is in perfect agreement with experimental data [1]. Both results hold true for simulations performed in periodic and fully non-periodic settings.

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