LOCALISED EXACT SOLUTIONS OF PIPE FLOW

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Abstract Turbulent transition in extended shear flows at low to moderate flow rates is subcritical and takes the form of localised patches of spatio-temporal complexity surrounded by otherwise laminar flow. We study numerically the laminar-turbulent boundary of pipe flow in long domains that allow for spontaneous localisation. We find that the dynamics on the boundary is governed by localised modulated travelling waves that share key structural properties with turbulent puffs.

INTRODUCTION

Turbulence is a common phenomenon in nature. Although the governing equations are well established and their numerical simulation reproduces the minutest details of turbulent motion, a theoretical understanding of the mechanisms behind transition remains elusive. An intriguing feature of low Reynolds number (Re) turbulent dynamics in extended domains is that it appears subcritically in the form of isolated patches surrounded by laminar flow.

A landmark problem for the study of subcritical transition in extended shear flows is the fluid flow down a straight pipe of circular cross-section [4]. For moderate values of the Reynolds number the laminar parabolic flow may undergo transition to turbulence following finite-amplitude perturbations despite remaining linearly stable. The computation in short periodic pipes of exact solutions that take the form of nonlinear travelling waves [1], combined with the analysis of the bifurcations they undergo [2] into increasingly complex flows, constitutes a decided step towards understanding the phenomena of subcritical transition and turbulence from a local perspective, but the study of localisation requires longer –more realistic– domains. Computations restricted to the laminar-turbulent boundary in long pipes have been shown to converge on a localised chaotic structure [3], but no underlying exact solutions have been reported.

LOCALISABLE TRAVELLING WAVES

Travelling waves come in a variety of symmetries. The most common is a combined shift-reflect symmetry (invariance to half a wavelength shift followed by reflection with respect to a certain diametral plane) that precludes localisation. By focusing on known travelling wave solutions that do not possess this symmetry, the potential for localisation as the domain is stretched remains intact.

As a matter of fact, numerical computations on the laminar-turbulent boundary using the edge-tracking algorithm [5] at moderate Re in short periodic pipes with two-fold azimuthal periodicity and reflection symmetry result in a travelling wave with the usual streak-vortex arrangement (figure 1). This travelling wave can be continued in Re as well as in κ

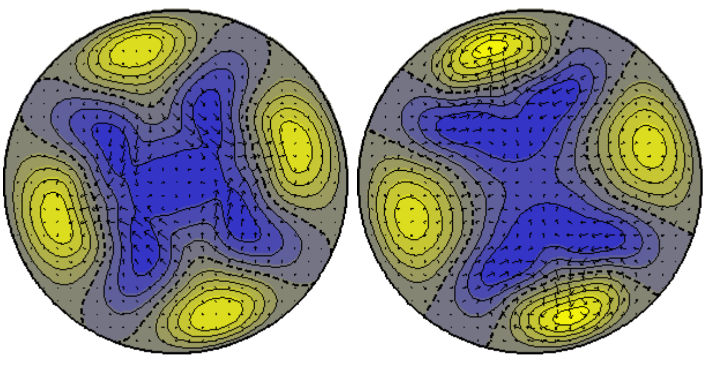

Figure 1. Two-fold azimuthally periodic and reflection-symmetric travelling wave at $Re = 2500$ in a periodic 5-diameter-long pipe. Perturbation field streamwise velocity contours at two axial locations separated by half the wavelength (yellow for positive, blue for negative).

(wavenumber) to unfold a full family that behaves the same way most travelling wave families do at sufficiently low Re. For a given κ they undergo a saddle-node bifurcation issuing an upper-branch of solutions with friction factors compatible with turbulent values. Fixing Re and varying κ the branch goes in a loop, so that solutions only exist within a limited range of wavelengths.

LOCALISED MODULATED TRAVELLING WAVES

The question then arises as to which solutions organise the laminar-turbulent boundary in pipes that are longer than the travelling waves maximum wavelength. The answer is that for sufficiently long pipes, edge-tracking converges on a branch of modulated travelling waves that are localised in the streamwise coordinate (figure 2). These waves core region

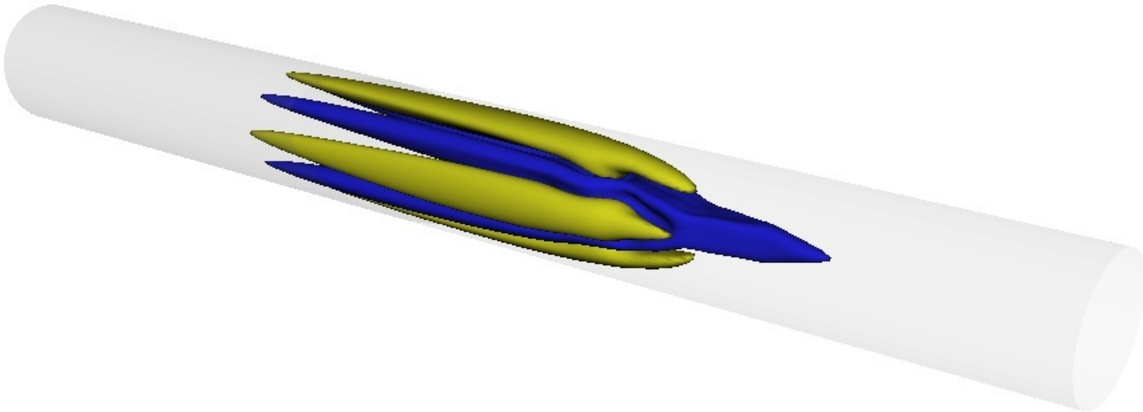

Figure 2. Two-fold azimuthally periodic, reflection-symmetric localised modulated wave at $Re = 2200$. Axial velocity isosurfaces (yellow for positive, blue for negative).

closely resembles the periodic travelling wave of shorter pipes, but their global structure is reminiscent of turbulent puffs. Continuation to lower Re reveals that the branch bends in a saddle-node and that, as progress is made on the upper-branch to larger Re, the properties of the wave approach those of puffs.
Similar solutions may be found with different azimuthal periodicities, pairing up with corresponding reflection-symmetric periodic travelling waves.

CONCLUSIONS

Localised modulated travelling waves seem to organise the laminar-turbulent boundary of long pipes in the same way travelling waves organise that of short periodic pipes. The discovery of such solutions is a key piece to understanding the phenomenon of localisation, a salient feature of shear flow turbulence, as well as of transition and turbulence in realistic situations.

References

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