JET FORMATION BY POTENTIAL VORTICITY MIXING AT LARGE AND SMALL SCALES

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Abstract We examine the formation of zonal jets in geostrophic turbulence with an emphasis on the inhomogeneous potential vorticity mixing by turbulent eddy and wave motions. Jet regimes are found to depend in a simple way upon two non-dimensional parameters, which may be related to the three natural length scales of the system: the Rhines scale, the forcing scale, and a length scale relating the strength of the forcing to the background potential vorticity gradient. Strong jet motions typically emerge when the forcing strength is weak. Furthermore, consideration of cases where the forcing acts at large scales provides an explicit demonstration that jet formation may be considered independently of the two-dimensional inverse energy cascade. Finally, two distinct types of potential vorticity mixing are identified, one dominated by turbulent eddies, the other through the action of localized critical layers.

A striking feature of the large-scale turbulent motions of atmospheres and oceans is the presence of well-defined zonal, or longitudinally aligned jets, coexisting with a background turbulent flow. The process of jet formation is understood most naturally in terms of the tendency for eddy and Rossby wave motions to mix potential vorticity, the fundamental dynamical quantity of the flow, whose background planetary gradient provides the restoring force for the Rossby waves themselves. Through the anisotropy of this background gradient, generation and dissipation of Rossby waves give rise directly to accelerations in the zonal direction: in particular, eddy fluxes of potential vorticity are related to the convergence of the eddy flux of zonal momentum through a generalized Taylor identity [7, 1, 2]. Eddy mixing of potential vorticity may thus be used as a general framework to describe jet formation in many situations, regardless of the length scales associated with dynamical forcings.

In this talk, we examine different jet/turbulence regimes in the simplest possible system of geostrophic turbulence on a mid-latitude beta-plane, using high-resolution, long-time numerical integrations. Two separate cases are considered, in which forcing scales are either (i) much smaller than, or (ii) comparable to the scale of the emerging jets. In the first case, the late-time distribution of potential vorticity is found to depend in a simple way on a single non-dimensional parameter, which may be conveniently expressed as the ratio $L_{\mathrm{Rh}}/L_\varepsilon$, where $L_{\mathrm{Rh}} = \sqrt{U/\beta}$ is the traditional Rhines scale [4], in which U may be taken as the root-mean-square velocity, and $L_\varepsilon = (\varepsilon/\beta^3)^{1/5}$ is a length scale relating forcing strength ε to planetary potential vorticity gradient β [3]. It is shown here that jet strength increases with $L_{\mathrm{Rh}}/L_\varepsilon$, with the limiting case of the potential vorticity staircase [2], comprising a monotonic, piecewise-constant profile in the north-south direction, being approached for $L_{\mathrm{Rh}}/L_\varepsilon \sim O(10)$ [5]. At lower values of $L_{\mathrm{Rh}}/L_\varepsilon$, eddies created by the forcing become sufficiently intense to continually disrupt the steepening of potential vorticity gradients in the jet cores, thus preventing strong jets from developing.

In the second case, in which the forcing scales, L_f are comparable to the scale of the emerging jets, the flow evolution depends on the ratios of all three length scales L_f, L_{Rh} and L_ε [6]. The potential vorticity is again found to organize into a piecewise constant staircase-like profile, monotonic in latitude, provided now that the additional requirement $L_{\mathrm{Rh}}/L_f \gtrsim 1$ is satisfied. That strong jets are observed even when $L_{\mathrm{Rh}} \approx L_f$ indicates, in particular, that jet formation may be considered completely independently from dynamical processes associated with the two-dimensional turbulent inverse energy cascade. More generally, the character of potential vorticity mixing is shown to depend on the ratio L_ε/L_f, occurring predominantly in localized critical layers when $L_\varepsilon/L_f \lesssim 1/6$, in contrast to the more uniform small-scale turbulent eddy mixing found when $L_\varepsilon/L_f \gtrsim 1/6$. In the former case, additional care must be taken with the form of the dynamical forcing to ensure that the material advection of potential vorticity is not obscured. Summarizing the overall dependence on all three length scales, a combined condition for the formation of strong zonal jets may be expressed as $L_{\mathrm{Rh}}/L_\varepsilon \gtrsim \max\{6, L_f/L_\varepsilon\}$.

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