LOCALIZED PERIODIC ORBITS IN PLANE POISEUILLE FLOW

Stefan Zammert & Bruno Eckhardt
Philipps-Universität Marburg

Abstract A family of localized periodic orbits for the plane Poiseuille system is presented. The discovered orbits exist in long but narrow domains for a wide range in Reynolds number and are localized in streamwise direction. In long and wide domains we were able to obtain fully localized periodic orbits. Furthermore, the orbits are localized version of the edge state in smaller domains that is a simple travelling wave.

INTRODUCTION

The study of exact solutions of the Navier-Stokes equation has attracted lots of attention within the past two decades. Starting with the discovery of the first exact 3-dimensional solution for plane Couette flow by Nagata [7], many exact solutions were found for various systems. For instance, Gibson et al. [4] studied in detail exact solutions in plane Couette flow, while Faisst et al. [2] and Wedin et al. [12] discovered the first travelling waves for pipe flow. In particular, two types of exact solutions receive special attention. One type are the so called *edge states* [11, 9]. These states have a stable manifold of co-dimension one which is able to divide state space. Edge states play an important role in the transition process and the formation of the turbulent attractor [6]. The second important class of exact solutions are localized ones. To use the tools of dynamical systems theory to explain the dynamics of localized structures such as puffs in pipe flow or turbulent spots in plane Couette or Poiseuille flow, localized exact solutions are necessary [5]. For the case of plane Couette flow Schneider et al. [10, 8] studied spanwise localized solutions and recently Avlia et al. [1] reported a streamwise localized periodic orbit in pipe flow. Nevertheless, until now there is a lack of fully localized solutions for plane shear flows that could help to understand the formation of turbulent spots. The focus of our research is on plane Poiseuille flow. For this system we investigate the edge state for different domains sized and are able to contribute exact solutions for the two important classes.

THE EDGE STATE IN SMALL DOMAINS

For all our simulations the *channelflow*-code [3] is used. We applied the method of edge tracking to the plane Poiseuille system and were able to identify the edge state. In small domains ($L_x = L_z = 2\pi$, $L_y = 2$) the edge state is a simple travelling wave. The travelling wave appears in a saddle-node bifurcation at *Re* ≈ 459 and can be continued to Reynolds numbers far above the critical Reynolds number. In figure 1a the average flowfield in the YZ-plane perpendicular to the direction of the flow is shown. It is evident that the wave fills the entire domain and is dominated by a strong low-speed streak. Furthermore, it is symmetric with respect to the midplane. For Reynolds numbers above 510 the wave has only one unstable direction and therefore it is an edge state.

LOCALIZED PERIODIC ORBITS

Using appropriate initial guesses for a Newton-method it is possible to find localized versions of the travelling wave in longer boxes. In a domain with extend $L_x = 32\pi$ and $L_z = 2\pi$ a streamwise localized periodic orbit is found. The period of the orbit is of the order of 50 time units, but the state only varies slightly within a period. In figure 1b the period depending on the Reynolds number is shown for the upper and the lower branch of the solution. As a visualization of the flowfield the streamwise velocity in the midplane at the time of minimal energy-norm is shown in figure 2b. It is obvious that the orbit has a strong low-speed streak which is surrounded by a larger high-speed streak. Comparing with the flowfields of the edge state travelling wave, which is shown in figure 2a, one recognizes some similarities between the two exact solutions. The similarity is even stronger if one compares the flowfield in the YZ-plane of the travelling wave to the flowfield in the core region (close to x=0 in figure 2b) of the periodic orbit. The periodic orbit appears in a saddle-node like bifurcation at *Re* ≈ 1000 and can be continued to *Re* > 3000. In wide domains it is possible to obtain periodic orbits that are localized in streamwise and spanwise direction. We will present and discuss the properties of the streamwise localized and the fully localized orbits depending on the Reynolds number. In particular, the stability and the strength of the localization depending on the Reynolds number is discussed. For example, it turns out that the localization is strongest for low Reynolds numbers and that the streamwise length of the localized states increases with the Reynolds number.

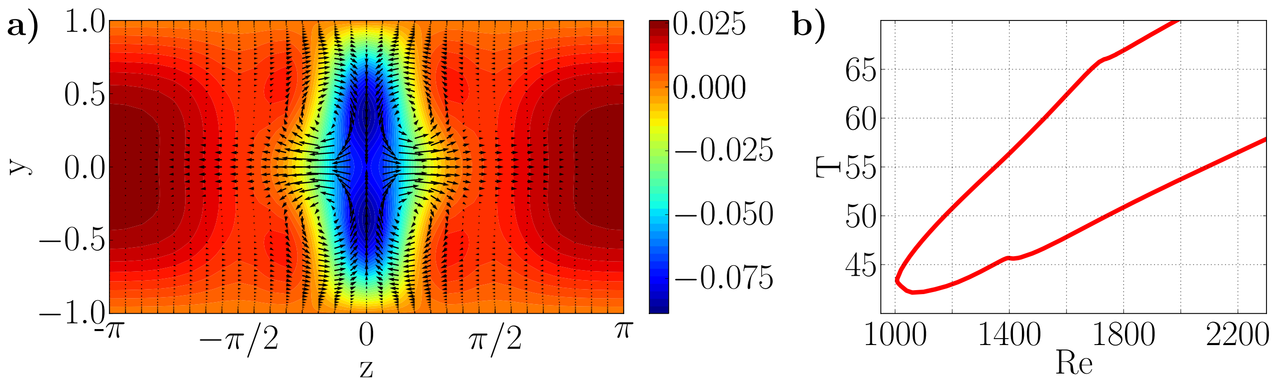

Figure 1. a) Average flowfield in the YZ-plane for the edge state travelling wave that was found in a domain of size $L_x = L_z = 2\pi$ at *Re* $= 1400$. The arrows indicate the in-plane velocity field and the colors the downstream component. b) Period of the streamwise localized periodic orbit vs. the Reynolds number. The domains size is $L_x = 32\pi$, $L_z = 2\pi$.

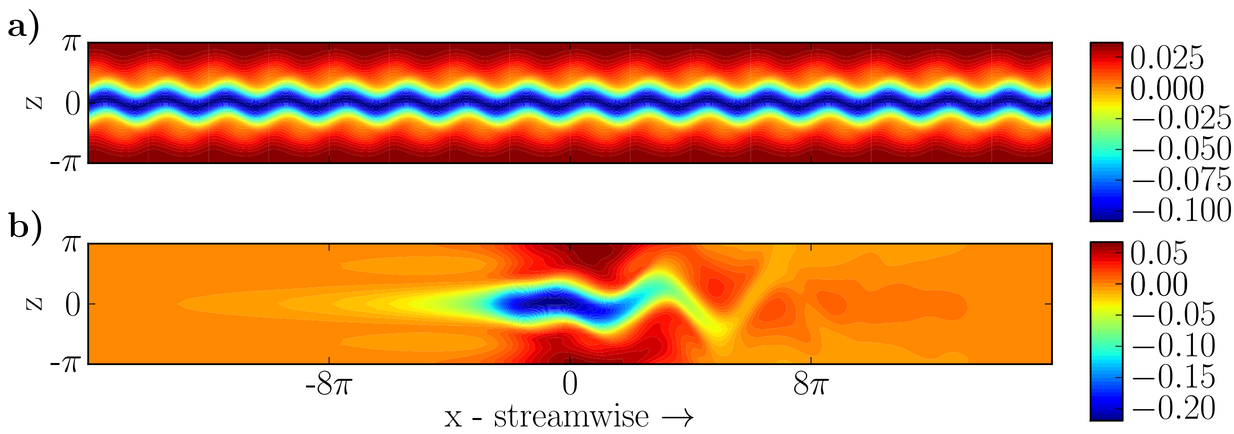

Figure 2. a) Instantaneous streamwise velocity (color coded) in the midplane for the edge state in the small domain with $L_x = L_z = 2\pi$ (periodically continued). b) Instantaneous streamwise velocity in the midplane for periodic orbit. For both plots the Reynolds number is 1050.

References

[1] M. Avila, F. Mellibovsky, N. Roland, and B. Hof. Building blocks of turbulence. *arXiv preprint arXiv:1212.0230*, pages 1–12, 2012.
[2] H. Faisst and B. Eckhardt. Traveling Waves in Pipe Flow. *Physical Review Letters*, **91**(22):1–4, 2003.
[3] J. F. Gibson. Channelflow: A spectral Navier-Stokes simulator in C++. Technical report, U. New Hampshire, 2012. `Channelflow.org`.
[4] J. F. Gibson, J. Halcrow, and P. Cvitanović. Equilibrium and travelling-wave solutions of plane Couette flow. *Journal of Fluid Mechanics*, **638**:243, 2009.
[5] G. Kawahara, M. Uhlmann, and L. van Veen. The Significance of Simple Invariant Solutions in Turbulent Flows. *Annual Review of Fluid Mechanics*, **44**(1):203–225, 2012.
[6] T. Kreilos and B. Eckhardt. Periodic orbits near onset of chaos in plane Couette flow. *Chaos*, **22**, 2012.
[7] M. Nagata. Three-dimensional finite-amplitude solutions in plane Couette flow: bifurcation from infinity. *J. Fluid Mech*, **217**:519–527, 1990.
[8] T. M. Schneider, J. F. Gibson, and J. Burke. Snakes and Ladders: Localized Solutions of Plane Couette Flow. *Physical Review Letters*, **104**(10):104501, 2010.
[9] T. M. Schneider, J. F. Gibson, M. Lagha, F. De Lillo, and B. Eckhardt. Laminar-turbulent boundary in plane Couette flow. *Physical Review E*, **78**(3):3–6, 2008.
[10] T. M. Schneider, D. Marinc, and B. Eckhardt. Localized edge states nucleate turbulence in extended plane Couette cells. *Journal of Fluid Mechanics*, **646**:441, 2010.
[11] J. Skufca, J. Yorke, and B. Eckhardt. Edge of Chaos in a Parallel Shear Flow. *Physical Review Letters*, **96**(17):5–8, 2006.
[12] H. Wedin and R. R. Kerswell. Exact coherent structures in pipe flow: travelling wave solutions. *Journal of Fluid Mechanics*, **508**:333–371, 2004.