STABILITY ANALYSES OF FLOW THROUGH AN ANEURYSM: STEADY AND PULSATILE FLOWS

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Abstract The aim of the present work is to study the changes in flow characteristics as a function of the height and width of a model abdominal aortic aneurysm (AAA) subjected to steady and pulsatile flows. Specifically, we present results from the changes in the distribution of the wall shear stress (WSS) and global stability analyses at different stages of development of an AAA. AAA's are characterised by a bulge about the vessel centerline [3] which is modeled using a gaussian function, that gradually straightens out to a uniform circular pipe in the upstream and downstream directions. The model configurations considered in the study are characterised by the bulge width W and the bulge height H (height of the bulge above the inlet radius), measured in units of inlet pipe diameter D. The global stability analyses were carried out on axisymmetric base flows on which non-axisymmetric perturbations are allowed to grow. The physical nature of instabilities are examined using an energy-transfer analyses and a qualitative description of the instability mechanisms are outlined.

STEADY FLOWS

Though the flow is pulsatile in the aorta, the analyses using steady flows are pertinent, as it helps in isolating the effect of the wall topography on wall shear stress (WSS) and flow instabilities, from the pulsatile nature of the flow, thereby reducing the complexity of the problem lending us to draw stronger conclusions. A parabolic velocity profile is imposed at the inlet. The dimensionless bulge height H is varied from 0.100 to 1.000, for four different values of the bulge width W (0.25, 0.50, 1.00, 2.00). The base flow consists of a core of relatively fast moving fluid traveling through the aneurysm, surrounded by an outer annulus of slowly recirculating fluid. With the increase in bulge height, multiple recirculating regions are observed. This can be seen in figure 1(b, c) in which the streamlines of the base flow are shown for $W = 0.5$ at two different values of H. The values of WSS within the aneurysm is found to be significantly smaller than those observed in a circular pipe without a bulge. They reach a maxima at the distal end of the bulge where a sharp change from low negative values to peak positive values are observed, with the maximum about 50% than that of the value observed in the circular section of the pipe.

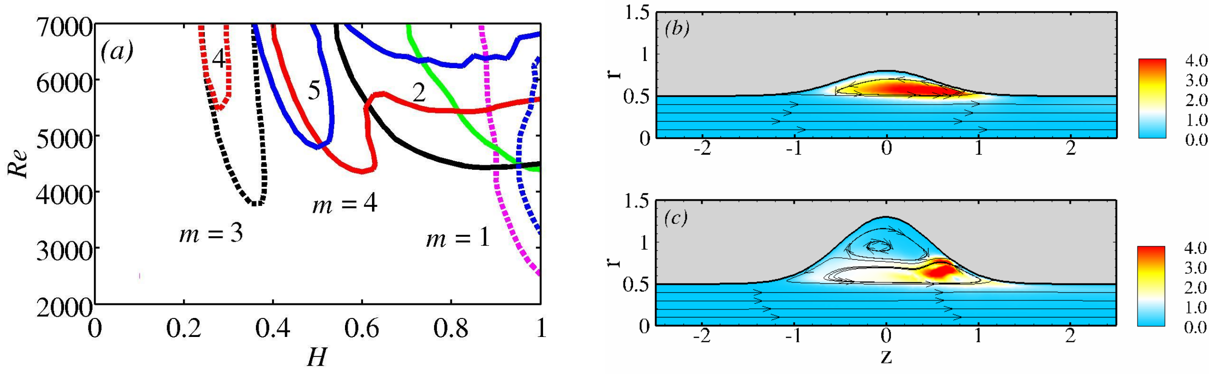

Figure 1. (a) Critical Reynolds number Re_c as a function of the bulge height H for different azimuthal mode numbers, m, at a $W = 0.5$. The stationary critical modes are denoted using dotted lines and the oscillatory modes using continuous lines. (b, c) The streamlines of the base flow and the amplitude of the global mode defined as the square-root of the energy density field. Parameter settings: (b) $W = 0.5$, $H = 0.3$, $m = 3$, $Re = 5500$, (c) $W = 0.5$, $H = 0.8$, $m = 3$, $Re = 5500$.

To the computed axisymmetric basic flow, three-dimensional perturbations were imposed that are homogeneous in the azimuthal direction and the resulting eigenvalue problem was solved. The stability analysis revealed that the critical modes are essentially confined to the bulge region, reaching their maxima at the downstream end of the bulge. This is shown in figure 1(b, c) in which global modes are visualized by the spatial distribution of their energy. Figure 1(a) shows the stability boundaries in the $H - Re$ plane for different azimuthal mode numbers m. For small values of bulge height, the aneurysm geometry can be viewed as a small perturbation to a circular pipe flow which is linearly stable at all Reynolds numbers. We can see that this is indeed the case as the flow is found to be stable to small values of H. As the bulge height is increased, the flow is found to become unstable to stationary critical modes shown by dotted lines in figure

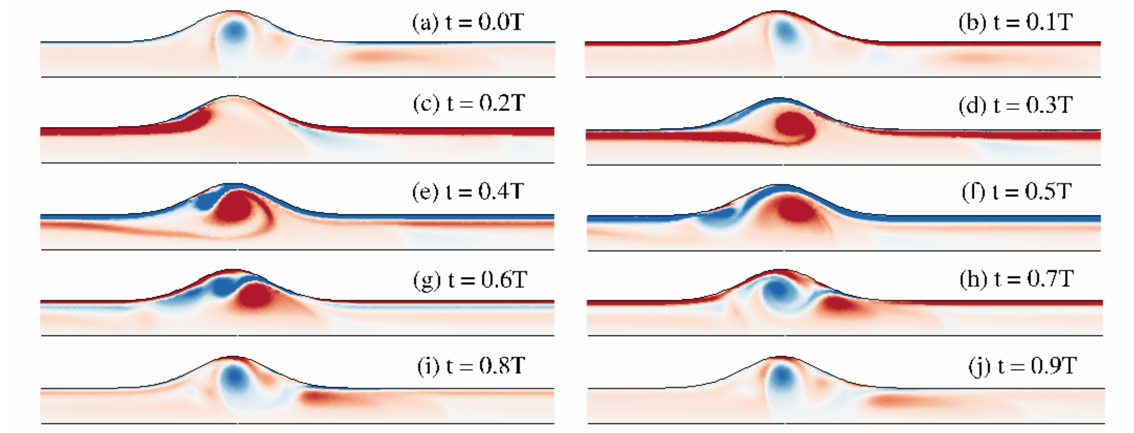

Figure 2. Dimensionless out-of-plane vorticity contours plotted at different time instants in a period. Parameter settings: $H = 0.45, W = 0.578466, \alpha = 10.7$.

1(a). For larger values of bulge height, the critical modes were found to be oscillatory shown by continuous lines in figure 1(a). Figure 1(b, c) shows the differences in the spatial distribution of energy between a stationary global mode at a small bulge height from that of an oscillatory global mode at a larger bulge height. The instability mechanisms were also found to be different in these different regimes, with a lift-up type of instability mechanism operating at small bulge heights and a centrifugal instability dominating at larger bulge heights. These were verified using an energy-transfer analysis similar to that found in [2, 1].

PULSATILE FLOWS

For the case of pulsatile flows, at the inlet a physiological waveform is applied found in the abdominal region of a human aorta that has been previously used in the literature [4, 5]. Computations were carried out at different bulge heights (0.15 - 0.70) and at different bulge widths (0.25 - 1.0). The Womersley number, α, that characterises the unsteady nature of the flow was varied (5 to 20) keeping the flow rate waveform the same. The flow dynamics was found to be completely different from the case of steady flows. Figure 2 shows the azimuthal vorticity component of the flow field at different time instants starting with the peak systole. Figure 2(a) shows the vorticity at the peak systole (peak forward flow). As the flow rate begins to decrease, frame (b), particles roll up into a vortex located just downstream of the middle of the aneurysm. This vortex enlarges through frame (c), where its anti-clockwise rotation couples with reversed flow in the tube to generate a weaker vortex of opposite rotational direction located near the proximal (upstream) end of the bulge. The secondary vortex persists in the bulge to frame (j), after which it is convected out of the bulge during the systolic phase of the pulse cycle. The WSS distributions were also studied at different time instants for different geometric and flow parameters.

Floquet stability analysis of the pulsatile flows is being carried out at present and would also be presented briefly during the conference.

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