POCKETS OF TURBULENCE IN PLANE COUETTE FLOW

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Abstract
We track the onset of chaos in plane Couette flow. The formation of a chaotic saddle through a boundary crisis is discussed. We show that inside the chaotic saddle, new saddle-node bifurcations create stable pockets of turbulence, which subsequently undergo boundary crisis. This leads to a non-monotonic variation of characteristic lifetimes with Reynolds number.

THE ONSET OF CHAOS

Studies of the transition to turbulence in linearly stable shear flows, such as pipe flow and plane Couette flow, have revealed a route to turbulence that passes through the formation of a chaotic saddle [1, 2]. We here present the analysis of the state-space structures and the bifurcations that lead to the creation of this saddle. The chaotic saddle supports long transients preceding sustained turbulence.

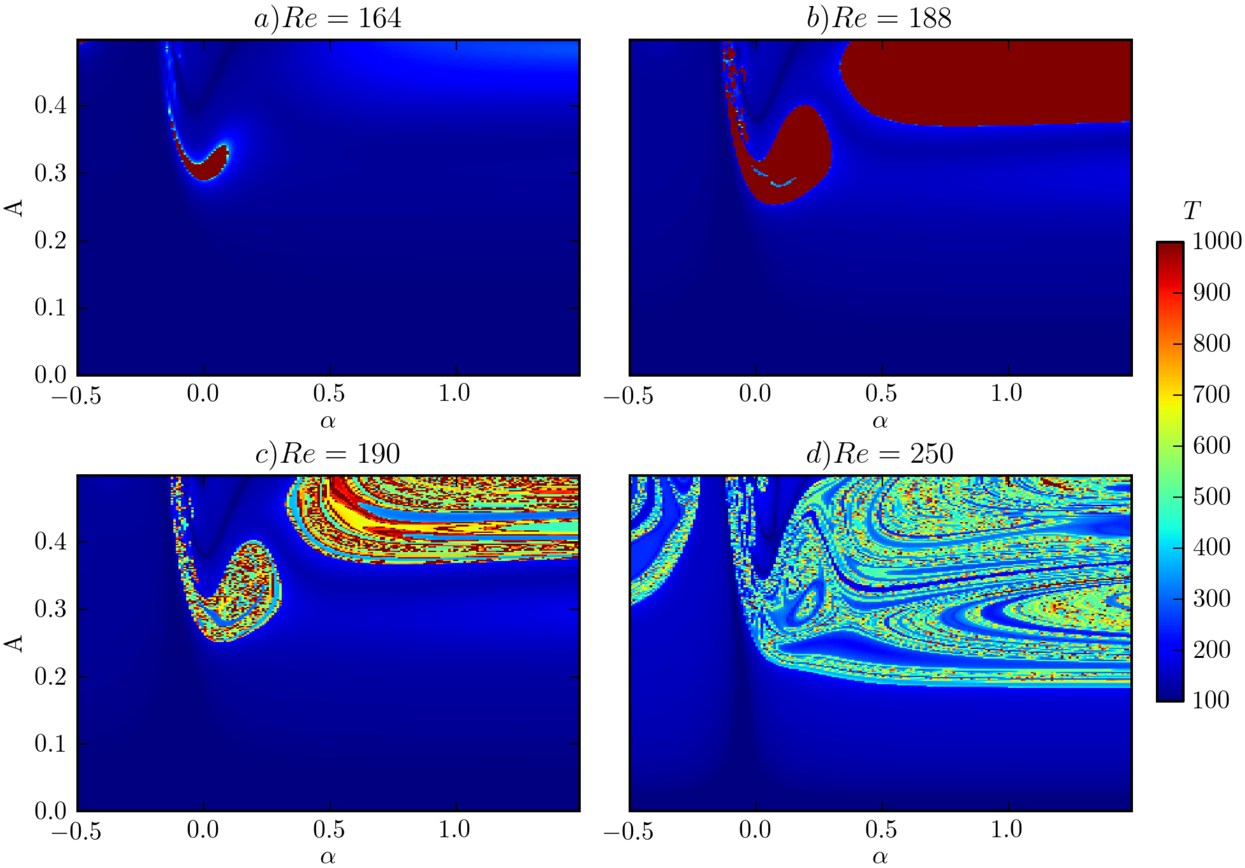

Figure 1. Two-dimensional slices of the state-space of transitional plane Couette flow. Coordinates are chosen such that the x-axis is an interpolation between the lower- and upper-branch Nagata-Busse-Clever states. The y-axis is the amplitude of the interpolated state. The structure of state-space is shown by the lifetimes T of initial conditions. (a) Just after the first saddle-node bifurcation. (b) Just before the boundary crisis. (c) Just after the boundary crisis. (d) At higher Reynolds number.

We study plane Couette flow in a small periodic domain of size $2\pi \times 2 \times \pi$ in the downstream, wall-normal and spanwise directions with imposed shift-and-reflect symmetry. In this cell, the Nagata state [4] is created as the lower-branch of a saddle-node bifurcation at Reynolds number $Re = 163.8$. It has exactly one unstable direction, while the corresponding upper-branch solution is stable. The latter one is hence an attractor that coexists with the laminar attractor. Figure 1(a) illustrates the situation by showing the lifetimes of initial conditions in a two-dimensional slice of the state-space at $Re = 164$. The small dark-red region corresponds to the basin of attraction of the upper-branch, the blue region to states that decay to the laminar state. The fixed-point undergoes a series of bifurcations that result in a chaotic attractor [3]. Its basin of attraction grows with increasing Re, figure 1(b).

At $Re = 188.7$, the chaotic attractor has expanded so much that it collides with the lower-branch state, leading to a boundary crisis. At this point, the basin of attraction opens up and the chaotic attractor turns into a chaotic saddle. By comparing the situation before (figure 1(b) at $Re = 180$) and after (figure 1(c) at $Re = 190$) the crisis, we still see the chaotic saddle as a region of rapidly varying lifetimes.

As Reynolds number increases further, the chaotic saddle grows to fill more and more parts of the state-space, figure 1(d) at $Re = 250$.

CREATING MORE CHAOS

Inside the chaotic saddle the distribution of lifetimes follows an exponential scaling, with a characteristic lifetime τ. Figure 2(a) shows τ as a function of Reynolds number, with each value of τ calculated from 50000 trajectories. Just above the crisis point in Re, the lifetime decays quickly. For higher Re it varies irregularly, before increasing more steadily for Re above 300.

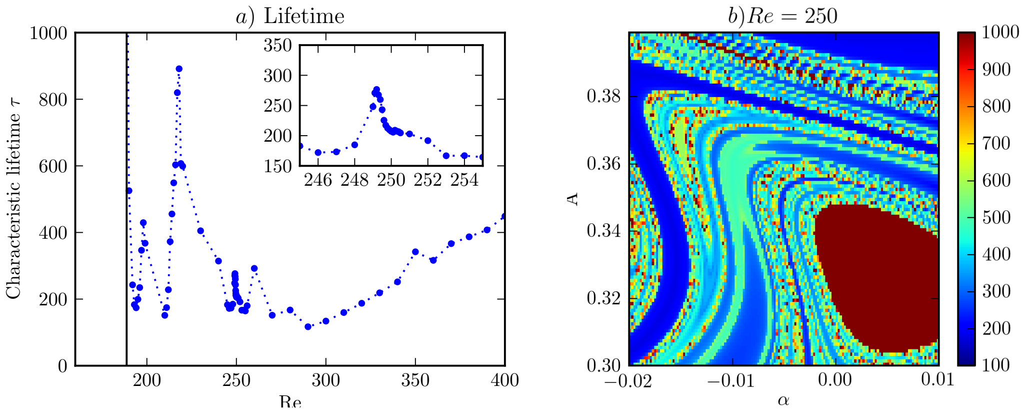

Figure 2. Non-monotonic lifetimes. (a) The characteristic lifetime as a function of Reynolds number varies non-monotonically. (b) A "pocket of turbulence", associated with a saddle-node bifurcation of periodic orbits, immersed in the chaotic saddle.

We are able to associate the non-monotic variation of lifetimes to the creation of "pockets of turbulence": in a small region in state-space a stable orbit and a hyperbolic point are created in a saddle-node bifurcation; this pocket grows, opens up and merges with other pockets, leading to an increase in lifetimes.

The crisis-bifurcation discussed above gives a first example of such a pocket. As a second example we study the peak around $Re = 250$, magnified in the inlet of figure 2(a). This peak can be related to a pair of periodic orbits which are created in a saddle-node bifurcation at $Re = 249.01$. In this bifurcation, the upper-branch orbit is an attractor, the lower-branch orbit has one unstable direction. The situation is hence similar to the one described above for the Nagata-solutions, only that now the saddle node bifurcation occurs inside the chaotic saddle. We see this chaotic pocket as the dark-red region in figure 2(b), where states never decay to the laminar state, immersed in the colorful saddle. The attractor is destroyed in a new crisis at $Re = 250.13$, leaving behind a larger chaotic saddle than before.

The creation of invariant solutions in saddle-node bifurcations and the destruction in crisis bifurcations provides one mechanism underlying the generally observed increase of turbulent lifetimes with Re. Note however that lifetimes can vary non-monotonically.

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