TEMPERATURE FLUCTUATIONS NEAR THE ULTIMATE-STATE TRANSITION IN TURBULENT THERMAL CONVECTION

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Abstract We report experimental results for temperature statistics in turbulent Rayleigh-Bénard convection (RBC) in a cylindrical sample of aspect ratio $\Gamma \equiv D/L = 1.00$ ($D = 1.12m$ is the diameter and $L = 1.12m$ the height). The measurements were conducted in the Rayleigh-number range $10^{13} \lesssim \mathrm{Ra} \lesssim 1.3 \times 10^{14}$ and Pr $\simeq 0.8$. Using the elliptic approximation (EA), we derived an effective Reynolds number Re_{eff} from temperature space-time cross-correlation functions $C_T(z, \tau)$. Results showed that $\mathrm{Re}_{eff} \propto \mathrm{Ra}^{\zeta_{eff}}$ with $\zeta_{eff} = 0.44 \pm 0.01$ for Ra $\lesssim 1.0 \times 10^{14}$. At Ra $\simeq 1.0 \times 10^{14}$ we found a transition in the Ra dependence of Re_{eff}. At the largest Re measured, near Ra $= 1.23 \times 10^{14}$, we found $E_T(k) \sim k^{-1.67}$ for the temperature energy spectra, which agrees well with the Obukhov-Corrsin $-5/3$ scaling for passive scalars in a turbulent flow at sufficiently large Reynolds numbers.

INTRODUCTION

Turbulent RBC is expected to undergo a transition from a “classical state" where the convection is driven by laminar boundary layers (BLs) to an ultimate state where the BLs become turbulent. The transition occurs when Ra exceeds a typical value Ra* [1]. The value of Ra* is expected to increase as Pr increases. For Pr $\simeq 1$, Ra* was estimated to be close to 10^{14} [2]. Above Ra*, the ultimate state is expected to be asymptotic in the sense that it will prevail as Ra diverges. Thus, above Ra* one can extrapolate laboratory measurements to astrophysically and geophysically relevant ranges of Ra, which often are well above 10^{20}. Recent experiments in Göttingen found Ra* in a laboratory-accessible Ra range using room-temperature compressed gas [3, 4, 5]. The results agreed well with calculations by Grossmann and Lohse (GL) [2]. Accompanied by this fundamental change in the BL state, interior bulk temperature fluctuations are expect to have different behavior from that observed in the classical state. That motivated us to conduct a further study of interior bulk-temperature statistics near the ultimate-state transition.

EXPERIMENTAL FACILITY AND MEASUREMENTS

The experiment was conducted with a large sample cell known as the High-Pressure Convection Facility (HPCF) which was located in an even larger pressure vessel known as the Uboot of Göttingen at the Max Planck Institute for Dynamics and Self-Organization in Göttingen, Germany. The sample was cylindrical with the diameter D and the height L both equal to $1.12m$, leading to the aspect ratio $\Gamma \equiv D/L = 1.00$. The top and bottom plates were made of aluminum. The sidewall was made of Plexiglas and had a wall thickness of 9.5 mm. Numerous thermal shields existed and were designed to prevent parasitic heat flow from the bottom plate to places other than through the sample [4, 5]. We used pressurized sulfur hexafluoride (SF_6) at temperatures close to ambient as the working fluid. Since for a perfect gas Ra $\propto P^2$ (P is the pressure) [4], we increased P up to the maximum allowed Uboot pressure of 19 bars in order to reach the highest Ra $\simeq 2 \times 10^{14}$.

We installed a vertical column of thermistors to measure temperature fluctuations at a radial distance of $1.5cm$ away from the side wall. The thermistor diameters were $0.36mm$. The vertical positions of the thermistors were distributed over a range of about 20 cm, nearly symmetrically about the mid-height of the sample. They were known with a precision of $1mm$. The sample was tilted by an angle $\beta \simeq 0.013$ rad relative to gravity so that measurements were conducted in the preferred rotation plane of an expected large-scale circulation (LSC). For each thermistor we used an ac bridge and a lock-in amplifier at a working frequency in the range $f_0 \simeq 1 \pm 0.4$ kHz to measure temperatures at a rate of 40 Hz.

From simultaneous temperature measurements for various vertical separations z, we calculated temperature space-time cross-correlation functions $C_T(z, \tau) \equiv \langle \delta T(x, t)\delta T(x + z, t + \tau)\rangle_t/(\sigma_1\sigma_2)$. Here τ is the time interval, δT the local temperature deviation from the mean value, and $\langle\rangle_t$ representing the averaging over time. $\sigma_i (i = 1, 2)$ is the temperature standard deviation at position i. For each Ra value, we used over 15-hour-long measurement (2×10^6 data points) to ensure adequate statistics. As temperature behaves like a locally passive scalar in the interior region of RBC, $C_T(z, \tau)$ were found to satisfy the elliptical approximation (EA) near the side wall, where the mean velocity U and the rms velocity V are comparable [6]. Then we can find a time delay τ_0, at which the measured auto-correlation $C_T(0, \tau_0)$ has the same value as the measured cross-correlation $C_T(z_0, 0)$. The relation between τ_0 and z_0 is given by $z_0 = \sqrt{U^2 + V^2}\tau_0$. By this means, we can obtained an effective velocity $V_{eff} \equiv \sqrt{U^2 + V^2} = z_0/\tau_0$ and the corresponding $\mathrm{Re}_{eff} = V_{eff}L/\nu$. Using the measured peak shift τ_p of $C_T(r_0, \tau)$, we can further obtain $U = V_{eff}^2\tau_p/r$ and $V = \sqrt{V_{eff}^2 - U^2}$ separately.

We defined the Reynolds numbers, $\mathrm{Re}_U \equiv UL/\nu$ and $\mathrm{Re}_V \equiv VL/\nu$, respectively.

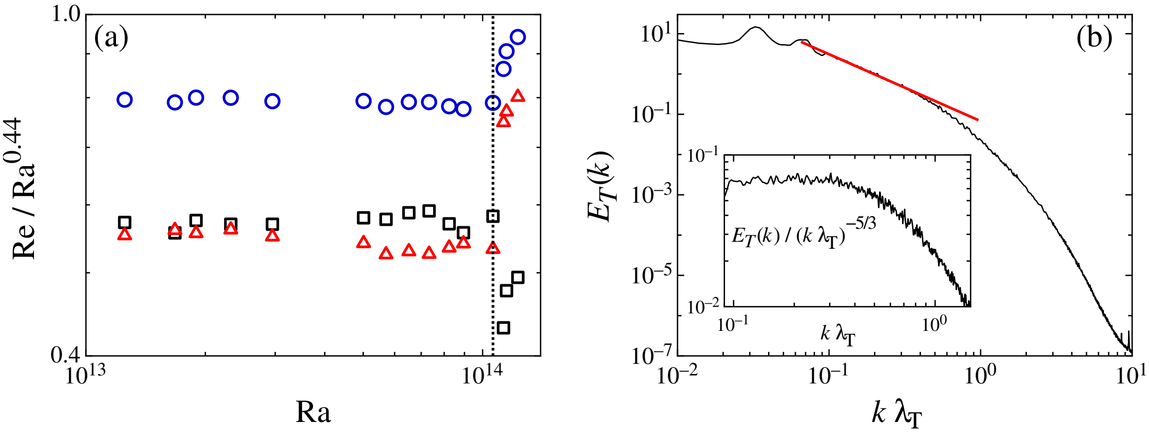

Figure 1. (a) Reduced Reynolds numbers $\mathrm{Re}_U/\mathrm{Ra}^{0.44}$ (squares), $\mathrm{Re}_V/\mathrm{Ra}^{0.44}$ (triangles) and $\mathrm{Re}_{eff}/\mathrm{Ra}^{0.44}$ (circles) as a function of Ra. The dotted line indicates $\mathrm{Ra} = 1.06 \times 10^{14}$. (b) The temperature energy spectrum $E_T(k)$ as a function of $k\lambda_T$ obtained for $\mathrm{Ra} = 1.23 \times 10^{14}$. The red line represents the power law $E_T(k) \sim (k\lambda_T)^{-1.67}$. The inset shows the compensated spectrum $E_T(k)/(k\lambda_T)^{-5/3}$ near the power-law range of $k\lambda_T$.

EXPERIMENTAL RESULTS

The results for the reduced Reynolds numbers $\mathrm{Re}_U/\mathrm{Ra}^{0.44}$ (squares), $\mathrm{Re}_V/\mathrm{Ra}^{0.44}$ (triangles), and $\mathrm{Re}_{eff}/\mathrm{Ra}^{0.44}$ (circles) are shown in Fig. 1 (a). For $\mathrm{Ra} \lesssim 10^{14}$, Re_U is described well by the scaling exponent 0.44, which agrees with the prediction from the GL model for the classical RBC state in a $\Gamma = 1$ sample [2]. Also we see that the fluctuation contribution Re_V is nearly equal to Re_U, and that also for Re_V we have $\mathrm{Re}_V \propto \mathrm{Ra}^{0.44}$. Consequently, also the effective Reynolds number Re_{eff} shows the same Ra dependence. For $\mathrm{Ra} \gtrsim 10^{14}$, we find that $\mathrm{Re}_U < \mathrm{Re}_V$, and thus that the fluctuations dominate Re_{eff}. Re_{eff} is seen to increase much more rapidly with Ra than it did below the transition. The drop of Re_U above $\mathrm{Ra} \simeq 10^{14}$ suggests a reduction of the LSC strength as Ra exceeds 10^{14}. Due to the limited Ra range, we were not able to determine scaling exponents for $\mathrm{Ra} \gtrsim 10^{14}$, but the data clearly indicate Ra dependences that differ from those of the classical regime. For the same sample, earlier measurements of the heat transport, expressed by the Nusselt number Nu, also showed a similar effect at $\mathrm{Ra} \simeq 10^{14}$. Results from both measurements indicate the onset of the transition to the ultimate RBC state.

In Fig. 1 (b), we show the temperature energy spectrum $E_T(k)$ as a function of the normalized wave number $k\lambda_T$. Here $E_T(k)$ was calculated from the temperature frequency power spectrum $P(f)$. Using the EA, one derives $E_T(k) = P(f)$ with $k = V_{eff}^{-1} f$ [6]. λ_T is the temperature Taylor microscale, which was calculated from fitting the equation $C_T(z,0) = 1 - (z/\lambda_T)^2$ to the data measured at small z. This measurement was conducted at the largest Ra (which was 1.23×10^{14}) because it spanned the longest range of $k\lambda_T$. In the range $0.09 \lesssim k\lambda_T \lesssim 0.3$, we found $E_T(k) \sim (k\lambda_T)^{-1.67}$. This scaling can be shown more clearly from the compensated plot of $E_T(k)/(k\lambda_T)^{-5/3}$ in the inset of Fig. 1b. The exponent -1.67 agrees well with the Obukhov-Corrsin $-5/3$ scaling for passive scalars in a turbulent flow at sufficiently large Reynolds numbers [7].

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