EFFECT OF COMPRESSIBILITY ON THE MERGING OF SHIELDED VORTICES

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Abstract The effect of compressibility on the critical merging distance of two co-rotating shielded vortices is studied using numerical simulations of two dimensional Euler equations. The simulations were carried out using a combination of sixth order compact finite differencing scheme, a tenth order compact filtering scheme and the classical Runge-Kutta scheme. Characteristic non-reflecting boundary conditions were applied on the domain boundaries. The results have revealed that the critical merging distance decreases as the vortex Mach number is increased.

INTRODUCTION

The phenomenon of merging of a pair of two dimensional co-rotating vortices is widely studied because of its importance in understanding physical mechanisms related to turbulent flows [1, 2, 3]. In this phenomenon there exists a critical merging distance above which merging does not occur. The critical merging distance for the merging of unshielded [1] and shielded [2, 4] vortices have been reported in the literature. A shielded vortex consists of a core with positive (negative) vorticity surrounded by a ring of negative (positive) vorticity. The interaction of a pair of co-rotating shielded vortices results in the formation of a tripole, if the initial separation distance is less than critical or in the formation of two self propagating dipoles, if the initial separation distance is greater than critical.

In the present work, the effect of compressibility on the critical merging distance of shielded vortices is studied. Sandham [5] had studied the effect of compressibility on the merging of unshielded vortices using direct simulations of Navier-Stokes equations. He found that for a given vortex Mach number, during the initial rotation phase, the separation distance between the vortices increased causing the rotation rate of the pair to decrease and the merging is delayed. This effect got magnified as the vortex Mach number increased, which he has defined in terms of circulation and the initial separation distance. This suggests that in case of shielded vortices, for a given initial separation distance just below critical, as the Mach number is increased, the vortices may not merge due to the effect of compressibility.

NUMERICAL SIMULATIONS

The interactions of two co-rotating shielded vortices have been simulated by numerically solving the Euler equations for two dimensional compressible flows. Effects of viscosity are not discussed here. The governing equations were non-dimensionalized with vortex core radius R_c, the maximum tangential velocity $(u_\theta)_{max}$ and the ambient density ρ_∞. The tangential velocity (u_θ), density (ρ) and pressure (p) profiles of the shielded vortex studied here are given as:

$$u_\theta = r\ exp\left[(1-r^2)/2\right], \qquad \rho = \left[1-(\gamma-1)(M_v)^2\frac{exp(1-r^2)}{2}\right]^{\frac{1}{\gamma-1}}, \qquad \frac{p}{\rho^\gamma} = \frac{1}{\gamma {M_v}^2}, \tag{1}$$

where (r,θ) are the cylindrical co-ordinates and M_v is the vortex Mach number based on maximum tangential velocity and ambient acoustic speed.

The numerical solutions were obtained by using a sixth order compact finite differencing scheme for spatial discretization along with a tenth order compact filtering scheme. The time integration was performed by using the classical fourth order Runge-Kutta scheme. The filtering was applied to the conservative variables after every ten Runge-Kutta time steps and the degree of filtering was chosen such that no well resolved scales get filtered. Characteristic non-reflecting boundary conditions were applied on all the sides of the square domain.

The simulations were initialized with two co-rotating shielded compressible vortices in a square domain of size $40R_c$ and resolved with 24 grid points per vortex core radius. The vorticity contours corresponding to the initial condition are shown in figure 1 for an initial separation distance, d equal to $2.5R_c$. The domain size and grid resolution were selected after carrying out a domain independence and grid independence study at $M_v = 0.7$ for $d = 2.3R_c$. Further simulations were carried out for different initial vortex separation distances ranging from $1.5R_c$ to $3.5R_c$ and for different vortex Mach numbers ranging from 0.1 to 0.7. M_v is varied by changing the ambient pressure. The results for lower M_v, (0.1 and 0.2) were consistent with the incompressible simulations of Valcke and Verron [4].

RESULTS AND DISCUSSION

The outcomes of all the simulations carried out in this study for different values of d and M_v are shown in figure 2 while the vorticity contours after a vortex circulation time $(2\pi R_c/(u_\theta)_{max})$ of 50 are shown in figure 3 for the same d, equal

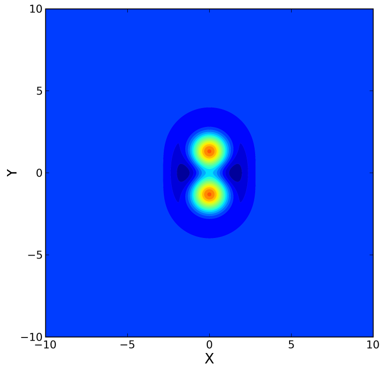


Figure 1. Vorticity contours of the two co-rotating shielded vortices in the initial condition. (Total 20 contours are plotted with vorticity varying from -1.0 to 4.0.)

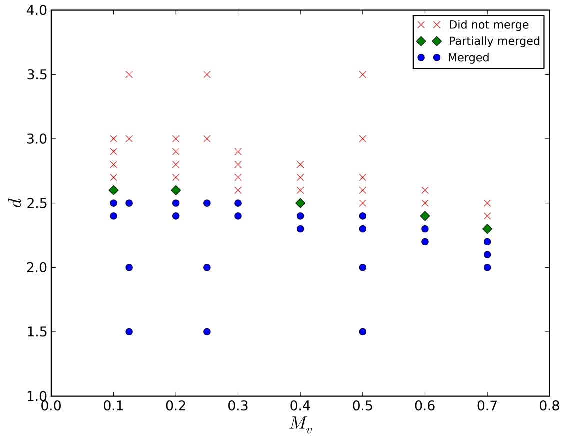
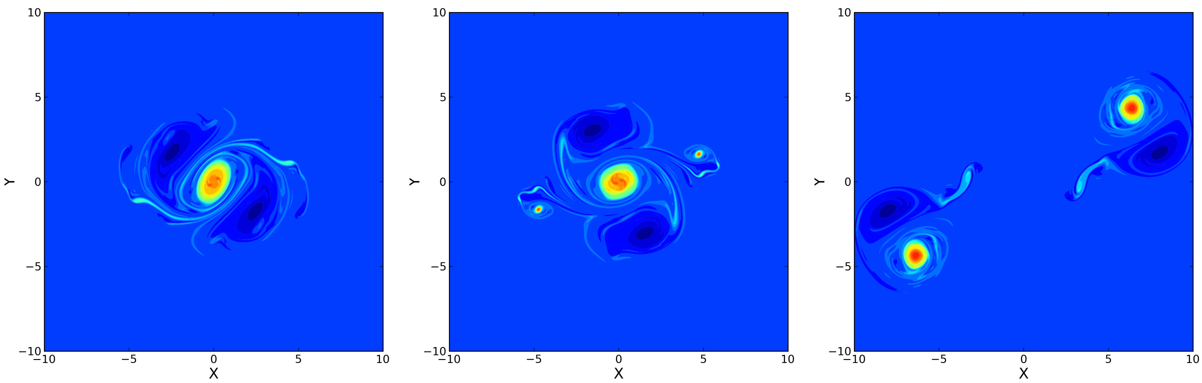


Figure 2. Outcomes of the interactions as a function of vortex Mach number and initial separation distance (in units of R_c).

to $2.5R_c$ and for increasing M_v. The value of d corresponding to figure 3 is just below critical for M_v equal to 0.3 and tripole formation is observed (figure 3a). When M_v was increased to 0.5 (figure 3c), as an effect of compressibility, the separation distance between the vortices had increased to such a value that caused the formation of two separate dipoles. Figure 3b shows partial merging of the vortices. In this case the effect of compressibility was not strong enough to prevent merging but still parts of the core of the resulting tripole got detached from it symmetrically. From this result it can be said that in order for the two co-rotating vortices to merge at M_v equal to 0.5, they should be kept closer to each other than at M_v equal to 0.3. Or in other words, the critical merging distance has decreased when M_v was increased from 0.3 to 0.5. Further, from figure 2 it can be seen that the critical distance for merging decreased monotonically with increasing M_v, up to M_v equal to 0.7.



(a) $M_v = 0.3$ (b) $M_v = 0.4$ (c) $M_v = 0.5$

Figure 3. Contours of vorticity after vortex circulation time of 50 with initial separation distance of $2.5R_c$.

More simulations are being carried out to study the interactions of co-rotating shielded vortices of unequal strengths. The results along with detailed physics will be presented during the conference.

References

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