ENERGY TRANSFER IN STRATIFIED TURBULENCE

Stefano Musacchio[1], Guido Boffetta [2] & Paolo Muratore-Ginanneschi [3]
[1]*Laboratoire J.A. Dieudonné , CNRS UMR 7351, Université de Nice Sophia Antipolis, Parc Valrose, 06108 Nice, France*
[2]*Dipartimento di Fisica and INFN, Università di Torino, via Pietro Giuria 1, 10125 Torino, Italy*
[3]*Department of Mathematics and Statistics, University of Helsinki PL 68, FIN-00014 Helsinki, Finland*

Abstract We investigate the mechanisms of transfer of kinetic and potential energy in stably stratified turbulent flows. By means of high-resolution direct numerical simulations and theoretical analysis of the Karman-Howarth-Monin relations we show that increasing the stratification the transfer of energy in the vertical direction is suppressed and a joint forward cascade of kinetic and potential energy develops.

Stably stratified flows are common in many instances ranging from geophysical flows to astrophysical applications (for a review see, e.g., [1, 2]). From a fundamental point of view, the consequences of stable stratification on turbulence and its effect on the energy cascade in two and three dimensional flows have been investigated by means of numerical simulations (see, e.g., [3, 4, 5, 6, 7]). In particular it has been found that stratification can induce a reduction of dimensionality and the emergence of large-scale quasi-two-dimensional structures (see, e.g., [8, 9]).

We investigate the mechanisms of transfer of kinetic and potential energy in a stably stratified turbulent flow with a mean vertical gradient $\bar{\rho}(z) = \rho_0 - \gamma z$. Within the Boussinesq approximation the equations for the velocity field $\boldsymbol{u}$ and the density fluctuations $\theta = \frac{N}{\gamma}(\rho - \bar{\rho})$ reads:

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \nu \Delta \boldsymbol{u} - N\theta \hat{e}_z + \boldsymbol{f} \qquad (1)$$

$$\partial_t \theta + \boldsymbol{u} \cdot \nabla \theta = \kappa \Delta \theta + N u_z \qquad (2)$$

where N is the Brunt-Väisälä frequency $N = (g\gamma/\rho_0)^{1/2}$, ν and κ represent molecular viscosity and diffusivity respectively and $\boldsymbol{f}$ is an external mechanical force. In the inviscid, unforced limit (1-2) conserve the total energy (kinetic plus potential) $E = E_K + E_P = \langle u^2 \rangle + \langle \theta^2 \rangle$.

We performed a series of direct numerical simulation of Eqs.1,2 at resolution $N_x \times N_y \times N_z = 1024 \times 1024 \times 128$ in a triply periodic domain of size $L_x = L_y = 2\pi$, $L_z = L_x/8$. The flow is sustained by a Gaussian white-in-time stochastic forcing localized in a narrow wave-number shell $|\boldsymbol{k}| \simeq k_f = 16$ with an energy input rate ε. At increasing the stratification, i.e. reducing the Froude number $Fr = (\varepsilon k_f^2)^{1/3}/N$, we observe a strong depletion of the transfer of kinetic energy in the vertical direction, and the formation of quasi-two-dimensional structures in the velocity field (see Fig. 1).

Remarkably, the spectral fluxes measured in our simulations indicates that such reduction of dimensionality of the flow is not sufficient to induce a transfer of turbulent kinetic energy toward large scales (as in the ideal case of two-dimensional turbulence) (see Fig. 2). The fluxes of kinetic and potential energy vanishes at $k < k_f$ even in the case of strong stratification.

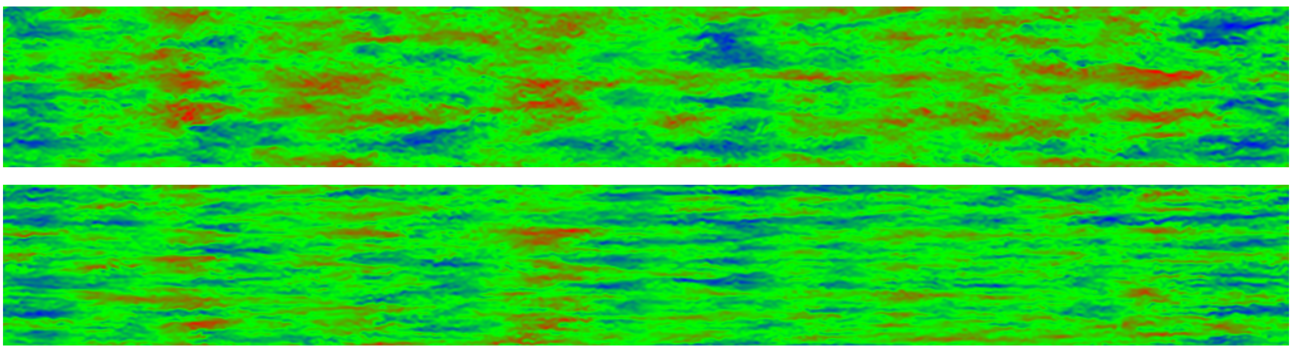
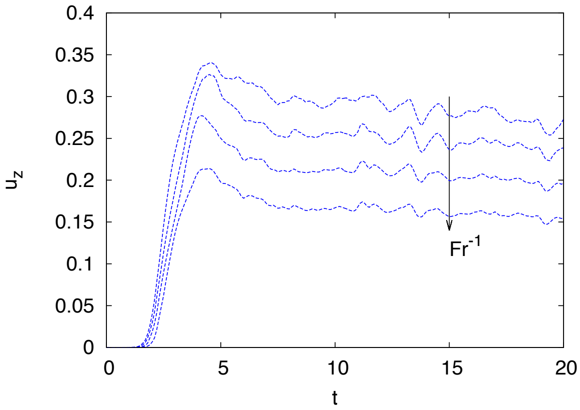


Figure 1. Vertical sections of the horizontal component u_x of the velocity field at $Fr = 0.14$ (top) and $Fr = 0.07$ (bottom).

In order to understand this phenomenon we perform a theoretical analysis of the Karman-Howarth-Monin relations generalized to the Boussinesq equations. The comparison between the theoretical predictions and the numerical findings indicates that the dynamics of stably stratified turbulent flows is characterized by a joint forward cascade of kinetic and potential energy toward small scales.



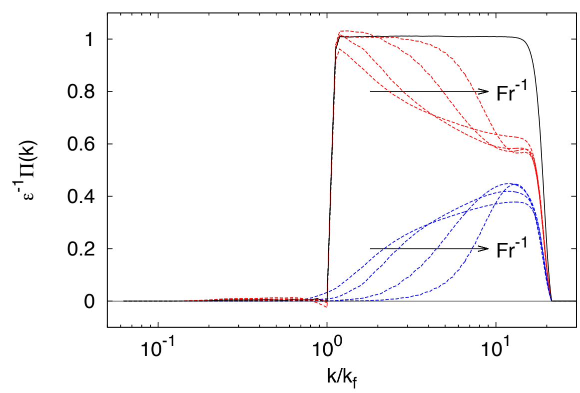


Figure 2. Left panel: Rms vertical velocity u_z for $Fr = 0.57, 0.28, 0.14, 0.07$. Right panel: Spectral fluxes of kinetic (red) and potential (blue) energy, normalized with the energy input ε. For comparison we show also the flux of kinetic energy in the case without stratification (black). No mean transfer of energy toward small wave-number $k < k_f$ is observed.

References

[1] Hopfinger E. J., *J. Geophys. Res.* **92**: 5287, 1987.
[2] Riley J. J. and Lelong M.-P., *Annu. Rev. Fluid Mech.* **32**: 613, 2000.
[3] Herring J. R. and Métais O., *J. Fluid Mech.* **202**: 97, 1989.
[4] Waite M. L. and Bartello P., *J. Fluid Mech.* **517**: 281, 2004.
[5] Lindborg E., *J. Fluid Mech.* **550**: 207, 2006.
[6] Brethouwer G., Brillant P., Lindborg E. and Chomaz J.-M., *J. Fluid Mech.* **585**: 343 2007.
[7] G. Boffetta, F. De Lillo, A. Mazzino and S. Musacchio, *Europhys. Lett.* **95**: 34001 2011.
[8] Smith L. M. and Waleffe F., *J. Fluid Mech.* **451**: 145, 2002.
[9] Métais O., Bartello P., Garnier E., Riley J. J. and Lesieur M., *Dyn. Atmos. Oceans*, **23**: 193, 1996.