INTERNAL STRUCTURE OF VORTICES IN SUPERFLUIDS AND IMPLICATIONS FOR QUANTUM TURBULENCE

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Abstract Dense Bose superfluids, as HeII, differ from dilute ones by the existence of a roton minimum in their excitation spectrum. It is known that this roton minimum is qualitatively responsible for density oscillations close to any singularity, such as vortex cores, or close to solid boundaries. We show that the period of these oscillations, and their exponential decrease with the distance to the singularity, are fully determined by the depth and the width of the roton minimum. We then study, based on a numerical simulation of the Gross-Pitaeviskii equation with a non-local interaction term, the implication of these oscillations on the vortex reconnection process, and quantify the amount of energy transfered from vortices to the background flow.

INTRODUCTION

HeII is the low temperature, low pressure superfluid phase of ^{4}He. The roton minimum, in its excitation spectrum, has been inferred by Landau from the viscosity measurements of Andronikashvili. It has been shown, by Feynman to be due to the dense packing of ^{4}He atoms. Solidification, which occurs above 2.5MPa for low temperature ^{4}He, can be seen as a condensation of rotons, due to their interactions.

The low value of the excitation energy at the roton minimum also suggests that the superfluid has a strong susceptibility for spatial perturbations of wave number k_o, the position of the roton minimum. Localized perturbations will then produce oscillations in the superfluid density in their neighborhood [1, 2, 3]. This has been known for long. However, the sensitivity of these oscillations to details of the real problem at hand is not clear.

Gross-Pitaevskii equations are often proposed for modeling the dynamics, and space dependence of the order parameter. The interaction term in these equations is chosen so as to fit the dispersion curve of elementary excitations. But a large class of such interaction terms can fit the same dispersion curve. What is the influence of their differences? What is the influence of details of the dispersion curve such as the phonon (long wavelength), maxons (maximum of the dispersion curve) or very short wavelength regions? Up to what precision do we have to fit the roton minimum (simple parabola, skewness)? At the end, it is known that, for strongly interacting superfluids as ^{4}He, density could not be simply proportional to the squared order parameter, as in dilute systems.

Determining the parameters which control the extension and amplitude of these oscillations, and discussing their implications are the goals of this paper. We shall focus on density oscillations far from the singularity, where they are small and amenable to a linear equation. We shall compare our approximation both with "exact" resolution of various Gross-Pitaevskii models, and with the results of first principle calculations.

The existence of density oscillations close to a vortex core is expected to have tremendous importance in the study of quantum turbulence in which a central role is played by vortex reconnections [4, 5] and thus it is useful to quantify how these oscillations depend on the shape of the roton minimum. In particular, this study could be necessary to the design of numerical investigations of vortex reconnections, as it was done formerly without taking into account the roton gap [6]. It is why we shall focus on the vortex problem.

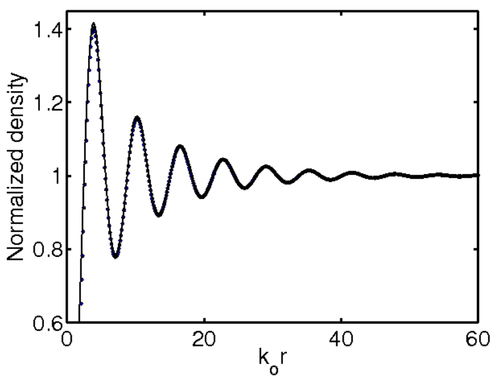

Figure 1. Comparison between the "equilibrium" density around a vortex (dots) in the non-local Gross-Pitaeviskii (Eq. 3), with the first order approximation Eq. 2 (full line)

RESULTS

We are interested in the deformation $\delta\rho(\vec{r})$ far from the perturbation. As $\delta\rho/\rho_o$ is small, it should obey a linear equation of the form $\mathcal{L}(\frac{\delta\rho}{\rho_o}) = V(\vec{r},t)$. Due to space and time translational invariance, the Fourier transform of this equation is local, i.e. $\tilde{F}(\omega,\vec{k})(\omega^2-\omega(\vec{k})^2)\frac{\tilde{\delta\rho}}{\rho_o}(\omega,\vec{k}) = \tilde{V}(\omega,\vec{k})$, which simply expresses that deformations can propagate in the superfluid, in the absence of any perturbation $\tilde{V}(\omega,\vec{k}) = 0$, if $\omega = \pm\omega(\vec{k})$. Our point is that $\tilde{F}$ is smooth in the neighborhood of $k = k_o$, as well as $\tilde{V}$, if the perturbation is localized. For a static perturbation, we can then write, in the general case:

$$\frac{\tilde{\delta\rho}}{\rho_o}(\vec{k}) = -\frac{\tilde{V}(\vec{k})/\tilde{F}(\vec{k})}{\omega(k)^2}\ . \tag{1}$$

In the case of interest for us, $\omega(k)^2$ has a deep minimum (the roton minimum) at $k = k_o$ that will clearly dominate the right hand side (RHS) of Eq. 1. Let us write in its neighborhood $\omega(k)^2 = \Omega^2 + c^2(k-k_o)^2$, where $\hbar\Omega$ is the energy roton gap and $m_* = \hbar\Omega/c^2$ the “roton mass”. Performing the integration in the complex plane, we get for the inverse Fourier transform, as explained in Ref. [7],

$$\frac{\delta\rho}{\rho_o}(r) \approx \sigma\frac{\exp(-k_1 r)}{\sqrt{k_o r}}\cos\left(k_o r - \pi/4 + \frac{k_1}{2k_o} + \phi_o\right) \tag{2}$$

with $k_1 = \Omega/c$, σ a constant and ϕ_o an additional phase depending on the precise shape of the perturbation. Let us stress that only ϕ_o depends on the precise shape of the perturbation, whereas oscillations wavelength $\sim 1/k_o$ and length scale of exponential decrease $\sim 1/k_1$ (i.e. the roton-gap width) come from the existence of a roton gap.

To show the realism of our prediction Eq. 2, we estimate numerically the fundamental solution in a cylindrical symmetry of a non local version of the Gross-Pitaevskii equation (see Ref. [7] for further details), namely:

$$i\hbar\partial_t\psi(\vec{r},t) = -\frac{\hbar^2}{2m}\Delta\psi(\vec{r},t) - \mu\psi(\vec{r},t) + \psi(\vec{r},t)\int d^3\vec{r}'\,\theta(|\vec{r}-\vec{r}'|/a)|\psi(\vec{r}',t)|^2 \tag{3}$$

where $\psi(\vec{r},t)$ is the order-parameter, $\theta(|\vec{r}-\vec{r}'|/a)$ represents the interaction potential between two atoms located at $\vec{r}$ and $\vec{r}'$, a is the range of the potential. The interaction is typically, as considered in Ref. [2], $\theta(\vec{x})$ is U_o if $|\vec{x}| < 1$, 0 for $|\vec{x}| > 1$ and μ is the chemical potential, fixing the equilibrium density $n = |\psi(\vec{r},t)|^2$. We show in Fig. 1 the comparison of our prediction to the density profile given by Eq. 3. We do reproduce accurately both oscillations our exponential decrease.

Thanks to this study, we understand and can predict the spatial extension of singularities in superfluids in the presence of a roton gap in the excitation spectrum located at k_o ($\sim \text{Å}^{-1}$) and of width $\sim 1/k_1$. We now design a numerical simulation solving Eq. 3 in a periodic cubic box of size $\sim 10\text{Å}^{-1}$. The initial condition is made of two orthogonal vortices (i.e. the product of two fundamental solutions of Eq. 3) as in Ref. [6] with furthermore the introduction of the roton minimum. As shown in Ref. [6], this leads to a reconnection. We quantify then the transfer of energy from the set of vortices to the background flow. Characterizing length scales, we are able to give a precise phenomenology of the transformation of vortex curvature energy into waves such as phonons and/or rotons.

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