SCALE-DEPENDENT STATISTICS OF LAGRANGIAN AND EULERIAN ACCELERATION IN ROTATING AND SHEARED HOMOGENEOUS TURBULENCE

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Abstract Direct numerical simulations of rotating and sheared homogeneous turbulence are performed in order to study the statistical properties of the Lagrangian and Eulerian accelerations. In particular, the influence of the Coriolis parameter to shear rate ratio f/S and the scale dependence of the statistics are investigated. The probability density functions (pdfs) of both Lagrangian and Eulerian acceleration show a strong and similar influence on the rotation ratio. The flatness further quantifies its influence and yields values close to three for strong rotation. For moderate and vanishing rotation, the flatness of the Eulerian acceleration is larger than that of the Lagrangian acceleration, contrary to previous results for isotropic turbulence. A wavelet-based scale-dependent analysis shows that the flatness of both Eulerian and Lagrangian acceleration increases as scale decreases. For strong rotation, the Eulerian acceleration is more intermittent than the Lagrangian acceleration, while the opposite result is obtained for moderate rotation.

INTRODUCTION

Acceleration statistics are of fundamental interest in turbulence ranging from theoretical questions to modeling of dispersion processes. For example, transport and mixing in geophysical flows play an important role for the spreading of pollutants in the atmosphere and the ocean. Numerous studies, either experimental due to the developement of advanced diagnostics [7] or numerical simulations enabled by the increasing power of super-computers, have been carried out. Recent reviews on Lagrangian properties of turbulent flows can be found in [8, 9].

The Lagrangian acceleration consists of two Eulerian components, $\partial u/\partial t$ and $u \cdot \nabla u$ where u denotes the velocity vector in an Eulerian reference frame. The local acceleration $\partial u/\partial t$ is also called Eulerian acceleration. It corresponds to the unsteady rate of change of the velocity vector, while the convective acceleration $u \cdot \nabla u$ contains effects due to spatial changes and includes nonlinear effects. The Lagrangian acceleration can be decomposed into irrotational parts, corresponding to the pressure gradient ∇p, and solenoidal parts, corresponding to the viscous dissipation term $\nu \nabla^2 u$. In the fully developed turbulent regime the pressure gradient is the dominant term and the viscous contribution can be neglected. For flows subjected to external body and surface forces, such as the Coriolis force for rotation or shear forces, additional terms have to be included which may alter the influence of the pressure gradient term.

Holzer and Siggia [3] have shown that for an incompressible Gaussian random velocity field the pressure probability distribution function (pdf) is negatively skewed and has exponential tails. From that purely kinematic result it can be deduced that the pressure gradient exhibits a Laplace distribution, as confirmed in [10]. Hence exponential tails of the Lagrangian acceleration should not be interpreted as a signature of intermittency, as discussed in [1] in the context of drift-wave turbulence.

Most studies of acceleration statistics focus on isotropic turbulence [8, 9]. It was found that the Lagrangian acceleration exhibits a strong intermittency which is reflected in the heavy tails of the pdfs. For example in [7] is was shown that particles undergo accelerations of up to 1,500 times the acceleration of gravity. Numerical simulations of isotropic turbulence confirmed these results [8]. Scale dependent statistics for turbulent flows using the orthogonal wavelet decomposition have been introduced in [2] and have been applied to study acceleration statistics for isotropic turbulence in [10].

The aim of the present work is to study acceleration in turbulent shear flow. Flows without rotation, with moderate rotation, and with strong rotation are considered, where the rotation configuration is either parallel or antiparallel. We analyze direct numerical simulation data published in [5, 6] and study the statistics of Lagrangian and Eulerian acceleration.

SAMPLE RESULTS

Figure 1 shows the normalized pdfs of Eulerian acceleration (left) and Lagrangian acceleration (right). Two families of pdfs are obtained: Gaussian like behavior for $f/S \pm 5$ and stretched exponential like behavior for the remaining cases. The non-normalized pdfs (not shown here) exhibit a strong and similar influence on the rotation to shear ratio f/S. We also find that, in all cases, the extreme values of the Eulerian acceleration are above the Lagrangian ones.

This influence can be further quantified by plotting the flatness of both accelerations as a function of f/S (Fig. 1, right). For strong rotation, i.e. $f/S = \pm 5$ we find flatness values close to 3, which confirm a Gaussian like behavior. For the remaining cases, larger values are observed with a maximum at $f/S = 0$ for a_E and f/S=0.5 for a_L. For positive values of f/S, the flatness of a_E is clearly larger than the one of a_L. The difference between both decreases with increasing f/S

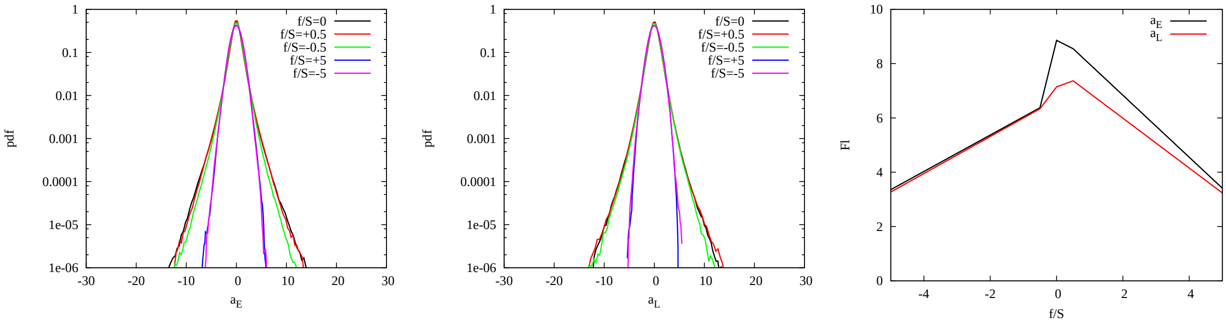

Figure 1. Normalized pdfs of Eulerian acceleration (left), normalized pdfs of Lagrangian acceleration (center), and flatness as a function of f/S (right).

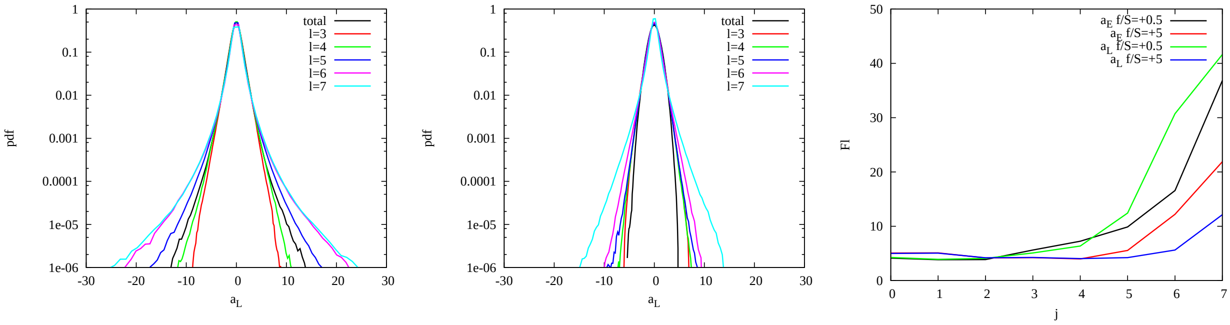

Figure 2. Scale-dependent normalized pdfs of Lagrangian acceleration for $f/S = +0.5$ (left) and $f/S = +5$ (right). Flatness as a function of scale for the two cases (right).

and for $f/S = \pm 5$ we find a value close to 3 for both. Contrary to what is found for isotropic turbulence [4], the Eulerian acceleration pdfs exhibit heavier tails than their Lagrangian counterparts. The fluctuating pressure gradients thus seem to be less important as in isotropic turbulence.

For the scale dependent analysis we focus on $f/S = +0.5$ and $+5$. The normalized pdfs of Eulerian and Lagrangian accelerations vary with scale for both cases. With decreasing scales, i.e. increasing j, the tails become heavier. Again we observe that heavier tails are present at all scales for the case $f/S = +0.5$. The scale dependent flatness of both the Lagrangian and Eulerian acceleration for the two cases shows a strong increase for decreasing scale (increasing j). For $f/S = 0.5$, the flatness of a_L is larger than the one of a_E (for $j > 4$), similar to what is observed for isotropic turbulence in [10]. In the case $f/S = 5$, we find that the values of a_E are larger than the values of a_L (for $j > 4$), which must be due to the fact that the effect of the nonlinear term becomes weaker for increasing rotation.

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