OPTIMAL TAYLOR-COUETTE TURBULENCE

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Abstract Strongly turbulent Taylor-Couette flow with independently rotating inner and outer cylinders with a radius ratio of $\eta = 0.716$ is experimentally studied. From global torque measurements, we analyse the dimensionless angular velocity flux $Nu_\omega(Ta, a)$ as a function of the Taylor number Ta and the angular velocity ratio $a = -\omega_o/\omega_i$ in the large-Taylor-number regime $10^{11} \lesssim Ta \lesssim 10^{13}$ and well off the inviscid stability borders (Rayleigh lines) $a = -\eta^2$ for co-rotation and $a = \infty$ for counter-rotation. With the help of laser Doppler anemometry, we provide angular velocity profiles and in particular identify the radial position r_n of the neutral line, defined by $\langle\omega(r_n)\rangle_t = 0$.

INTRODUCTION

Taylor-Couette (TC) flow (the flow between two coaxial, independently rotating cylinders) is, next to Rayleigh-Bénard (RB) flow (the flow in a box heated from below and cooled from above), the most prominent 'Drosophila' on which to test hydrodynamic concepts for flows in closed containers. For outer cylinder rotation and fixed inner cylinder, the flow is linearly stable. In contrast, for inner cylinder rotation and fixed outer cylinder, the flow is linearly unstable thanks to the driving centrifugal forces.

There are three control parameters in the system: the radius ratio $\eta = r_i/r_o$, the inner and outer cylinder Reynolds numbers Re_i and Re_o. Alternatively, these control parameters can also be expressed as: Taylor number Ta, the ratio of the angular velocities $a = -\omega_o/\omega_i$, and the radius ratio η. One global response parameter is the dimensionless torque G as the transport quantity or alternatively 'Nusselt number' (Nu) [1]. Another global response parameter is the degree of the turbulence in the gap, which is measured by the wind Reynolds number Re_w [2].

The aim of the present work is to study the angular velocity flux $Nu_\omega(Ta, a)$ as a function of a for given Taylor numbers. In addition, we provide laser Doppler anemometry (LDA) measurements of the angular velocity profiles $\langle\omega(r)\rangle_t$.

EXPERIMENTS AND RESULTS

The experiments are performed in Twente Turbulent Taylor-Couette (T3C) system, which has an inner cylinder with a radius of $r_i = 0.200$ m, a transparent outer cylinder with an inner-radius of $r_o = 0.279$ m, resulting in a gap-width of $d = r_o - r_i = 0.079$ m and a radius ratio $\eta = r_i/r_o = 0.716$. The height is $L = 0.927$ m implying an aspect ratio of $\Gamma = L/(r_o - r_i) = 11.7$. More details regarding the experimental facility can be found in ref. [3, 4].

We measure the dimensionless angular velocity flux $Nu_\omega(Ta, a)$ as a function of the Taylor number Ta and the angular velocity ratio $a = -\omega_o/\omega_i$ in the large-Taylor-number regime $10^{11} \lesssim Ta \lesssim 10^{13}$ and well off the inviscid stability borders (Rayleigh lines) $a = -\eta^2$ for co-rotation and $a = \infty$ for counter-rotation. We analyse the data with the common power-law ansatz for the dimensionless angular velocity transport flux $Nu_\omega(Ta, a) = f(a)Ta^\gamma$, with an amplitude $f(a)$ and an exponent γ.

The data are consistent with one effective exponent $\gamma = 0.39 \pm 0.03$ for all a as shown in Figure 1(a). The amplitude of the angular velocity flux $f(a) \equiv Nu_\omega(Ta, a)/Ta^{0.39}$ versus a is shown in Figure 1(b). It is found that the amplitude f(a) is maximal at slight counter-rotation, namely at an angular velocity ratio of $a_{opt} = 0.33 \pm 0.04$, i.e. along the line $\omega_o = -0.33\omega_i$. This value is theoretically interpreted as the result of a competition between the destabilizing inner cylinder rotation and the stabilizing but shear-enhancing outer cylinder counter-rotation.

To study this optimal transport, we directly measure angular velocity profiles and in particular identify the radial position r_n of the neutral line, defined by $\langle\omega(r_n)\rangle_t = 0$ for fixed height z. For these large Ta values the ratio $a \approx 0.40$, which is close to $a_{opt} = 0.33$, is distinguished by a zero angular velocity gradient $\partial\omega/\partial r = 0$ in the bulk. While for moderate counter-rotation $-0.40\omega_i \lesssim \omega_o < 0$, the neutral line still remains close to the outer cylinder and the probability distribution function of the bulk angular velocity is observed to be monomodal. For stronger counter-rotation the neutral line is pushed inwards towards the inner cylinder; in this regime the probability distribution function of the bulk angular velocity becomes bimodal, reflecting intermittent bursts of turbulent structures beyond the neutral line into the outer flow domain, which otherwise is stabilized by the counter-rotating outer cylinder [5].

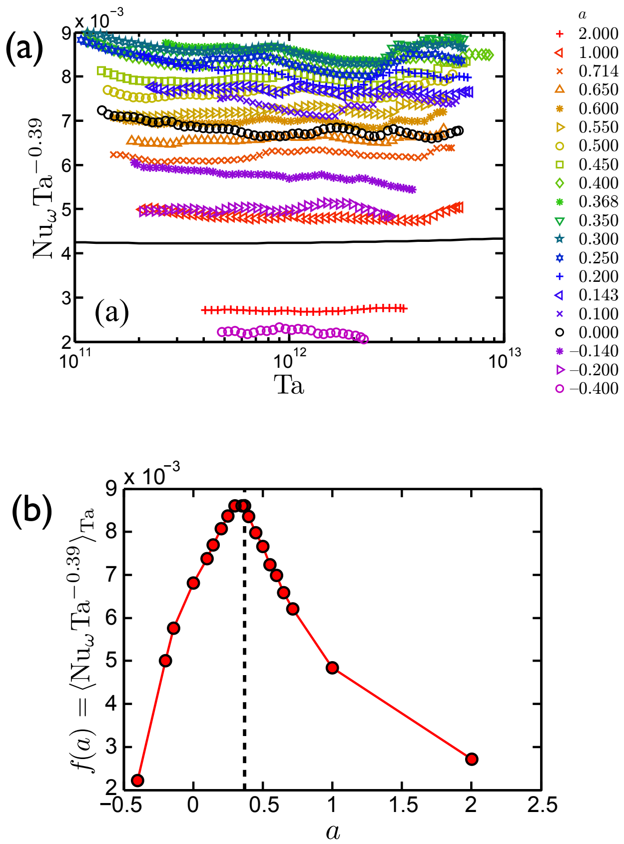

Figure 1. (a): $Nu_\omega(Ta, a)$ – compensated by $Ta^{0.39}$ – for various a as a function of Ta, revealing effective universal scaling. (b) Amplitude f of the effective scaling law $Nu_\omega \propto Ta^{0.39}$ as function of a. The dashed line corresponds to the optimal turbulence.

SUMMARY

In conclusion, we have experimentally explored strongly turbulent TC flow with $Ta > 10^{11}$ in the co- and counter-rotating regimes. We find that in this large Taylor number Ta regime and well off the instability lines the dimensionless angular velocity transport flux within experimental precision can be written as $Nu_\omega(Ta, a) = f(a) \cdot Ta^\gamma$ with either $\gamma = 0.39 \pm 0.03$ universally for all a. The transport is most efficient for the counter-rotating case along the diagonal in phase space with $\omega_o \approx -0.33\omega_i$

References

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