NUMERICAL INVESTIGATION ON TRANSITION OF 2-D FARADAY WAVES

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Abstract A binary-fluid direct numerical simulation of the spacial two dimensional Faraday waves is carried out with phase-field method. The period-tripling state is observed in our simulation at the transition region from basic standing waves to chaotic behaviors. We make the phase diagram and investigate the transition region in order to understand a mechanism of the period tripling.

INTRODUCTION

Faraday waves are known as an excellent example of pattern formations which occur typically in weakly nonlinear regimes. Meanwhile, turbulent states of Faraday waves exhibit many interesting properties [4],[6]. For example, the velocity fluctuations on the fluid surface are known to be similar to those of the two dimensional (2-D) dual cascade turbulence (the energy inverse cascade and the enstrophy direct cascade) [6]. How the pattern-formation state and the turbulent states are connected? This is our long term goal. In this paper, we address this question for two-dimensional Faraday waves which are amenable to detailed numerical simulations.(Note that a full simulation of 3-D Faraday waves become available as recently as in 2009 [3].)
Transition processes from a basic, standing-wave state of a linear neutral mode to a chaotic state (or a near wave-breaking state) of quasi 2-D Faraday waves have been studied experimentally [2],[1]. Between the two states (by increasing the vertical vibration amplitude), a peculiar state is found. This is called period-tripling state, in which the period of the wave is namely tripled almost suddenly. Its underling mechanism is not known. We investigate the transition including the period-tripling state by a direct numerical simulation with a phase-field modeling of the fluid interface.

NUMERICAL METHOD

The governing equations for the immiscible binary fluids in the present study can be written as

$$\nabla \cdot \boldsymbol{u} = 0, \quad \rho D_t \boldsymbol{u} = -\nabla p + \rho \boldsymbol{G} + \nabla \cdot \eta (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T) + \boldsymbol{s}, \tag{1}$$

where $\boldsymbol{G}$ is the acceleration force in the reference frame of the container $\boldsymbol{G} = (-g + A\cos(\omega t))\boldsymbol{e}_z$. Here, g, A, ω, ρ, η and $\boldsymbol{s}$ are respectively the acceleration of the gravity, the vibration amplitude, the angular frequency of the vibration, the density, the viscosity and the surface force. The fluid interface is handled with phase-field method. In this method, the phase-field ϕ specifies each fluid. In the fluid 1, ϕ equals to 1. In the fluid 2, ϕ equals to -1. In the interface, ϕ is under the condition$-1 < \phi < 1$. The phase-field ϕ is governed by the Cahn-Hilliard equation

$$D_t \phi = \kappa \nabla^2 \mu, \quad \mu = \frac{3\sigma\epsilon}{2\sqrt{2}} \left(-\nabla^2 \phi + \frac{\phi^3 - \phi}{\epsilon^2} \right), \tag{2}$$

where ϵ and κ are the model parameters of the phase-field method and σ is the surface tension. In this method, the surface force term $\boldsymbol{s}$ in Eq.(1) is $-\phi\nabla\mu$. The two fluids in the two dimensional domain are bounded by horizontal flat plates. At top and bottom plates $z = L_z, 0$, non-slip boundary condition is imposed. In the other direction, the periodic boundary condition is imposed at $x = 0, L_x$. In this study, we used L_x in the range from 1.46×10^{-4}m to 1.80×10^{-4}m and $L_z = 2.31 \times 10^{-4}$m with 128×128 grid points. We validated the phase-field method by comparing calculation results with linear analysis. Details of the discretization and validation are in the Takagi et al.[5].

RESULTS (PERIOD-TRIPLING STATE)

The period-tripling state was observed in quasi-two dimensional experiments by Jiang et al.[2] and Das et al.[1]. The important feature is that the state is observed in the transition to a chaotic state. In this state, the period of the standing waves are three times basic-period of Faraday waves. The basic period, here, is $T_b = 2T_v$ in the subharmonic case, where $T_v = 2\pi/\omega$ is the vibration period. Thus, the period of the period-tripling state is $3T_b$.
The numerical result (Fig.2) shows that the period of the interface elevation becomes $3T_b$ (the three times basic-period). Note that, because of numerical limitation, the physical parameters used in the simulation such as ρ, η, σ and so on are different from the experiments. Nevertheless we observed the similar behavior suggesting its universality.[5] In order to further investigate this states, we changed the wave number of the initial perturbation and the vibration amplitude with the numerical simulation.
Figure.1-(a) shows a phase diagram. The vertical axis p is nondimensional dumping parameter. The horizontal axis q is nondimensional forcing parameter. We investigated in detail the transition region from the basic-period state shown in

Fig.1-(b) to the chaotic state shown in Fig.1-(c). We expected a simple structure, i.e. there is only the period-tripling state in the region, but the expectation is not right. Namely, three states are observed, which are the period-tripling state, the modulation states (Fig.1-(d)) and the quasi-tripling state (Fig.1-(e)). In the modulation state, the behavior of interface is nearly basic-period state, but the behavior has modulation. In the quasi-tripling state, the behavior of interface is similar to the period-tripling state, but the period of this state does not equal to $3T_b$. As vibration amplitude is increased gradually more than the case of period tripling state in the transition region, the state becomes not chaotic but nearly basic periodic state again. Temporal Fourier analysis of the interface elevation confirms this picture. In Fig.2, we show the squared amplitude of the period-tripling mode (Fourier modes with the frequency $\frac{1}{3}\left(\frac{\omega}{2}\right)$ as a function of the normalized vibration amplitude q. The basic and chaotic states correspond to $q \leq 2$ and $q \geq 2.18$, respectively. In the transition region, $2 < q < 2.18$, the period-tripling mode is not kept necessarily large. In particular it nearly vanish for $2.1 \leq q < 2.18$, where the modulation state appears. In other words, there is a modulation-state window between the period tripling state and the chaotic state.

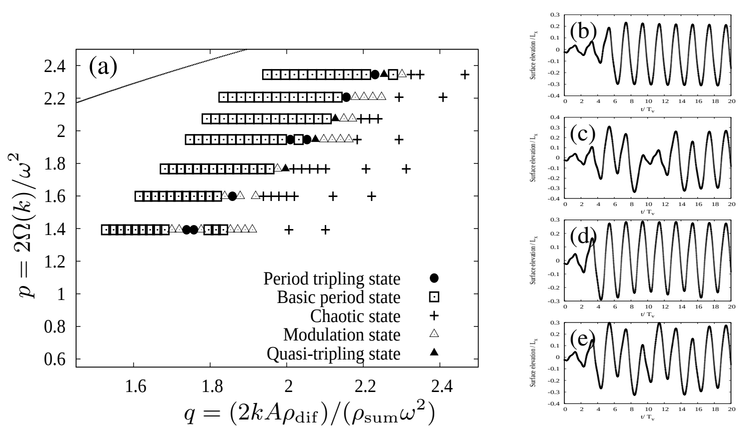

Figure 1. (a) the phase diagram of period-tripling. Here, $\Omega^2(k) = kg\{\rho_{\rm dif}/\rho_{\rm sum} + \sigma k^2/(g\rho_{\rm sum})\}$, where $\rho_{\rm sum}$, $\rho_{\rm dif}$ and k are ,respectively, the sum of the two densities, the difference of the two densities and wave number of the initial perturbation. We show typical behaviors (b) the basic-period state, (c) the chaotic state, (d) the modulation state, (e) the quasi-tripling state.

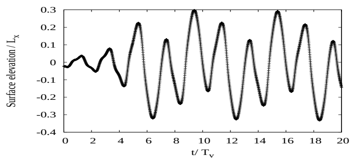

Figure 2. Temporal elevation of interface for period-tripling state at the $L_x/2$

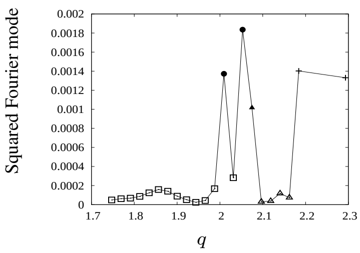

Figure 3. Fourier mode of the frequency $\omega/6$ corresponding to the tripled period as a function of the normalized vibration amplitude q for a fixed normalized vibration frequency $p = 1.95$.

SUMMARY · DISCUSSION

We executed numerical simulation with the phase-field method and reproduced the period-tripling state. The phase diagram of this state shows that period tripling state is in the transition region between the regions of the basic-period and the chaotic states. We can consider the following scenario about the period-tripling mechanism. As the vibration amplitude is increased, the other modes different from the basic-period mode become large and the transition from the basic-period state to the modulation state occurs. The interaction among these modes generates the period-tripling mode. The scenario is true at $p = 1.4,\ 1.6$ in Fig.1-(a). However, the scenario may not be true at large p because transition from the basic-period state to the period-tripling state occurs without the modulation state. Therefore, these modulation modes may be not needed essentially for the period-tripling state. In future, we investigate the nonlinear interaction in the wavenumber-frequency space in order to understand generation of the period tripling modes.

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