# Breakdown of Kolmogorov's scaling in grid turbulence

Lyazid Djenidi[1], Robert A. Antonia[1] & Sedat Tardu[2]
[1]*School of Engineering, University of Newcastle, Newcastle, Australia*
[2]*LEGI, University of Joseph Fourier, Grenoble, France*

*Abstract* A direct numerical simulation (DNS) based on the lattice Boltzmann method is carried out for decaying grid turbulence at low Reynolds numbers with the view to investigating possible departures from Kolmogorov scaling. 1D and 3D spectra show that the Kolmogorov scaling is no longer valid when the Reynolds number falls below a certain value. The results are in agreement with the low Reynolds number DNS in a 3D periodic box by Mansour and Wray [1]. We are now investigating possible departures from local isotropy when the Kolmogorov scaling breaks down.

## Introduction and Numerical Set Up

The first similarity hypothesis of Kolmogorov (or K41) implies that spectra of velocity fluctuations scale on the kinematic viscosity $\nu$ and the turbulent kinetic energy dissipation rate $< \varepsilon >$ at large Reynolds numbers. However, evidence, based on both DNS data and measurements, points to this scaling being also valid at small Reynolds numbers, provided effects due to inhomogeneities in the flow are negligible. Recently, Djenidi and Antonia [2] exploited this to develop a spectral method for estimating $<\varepsilon>$ in various turbulent flows. One can however expect that this scaling will break down when the Reynolds number becomes relatively small. Mansour and Wray [1] showed that for 3D periodic box turbulence, the energy power spectrum scaled on Kolmogorov variables deviates for the "universal Kolmogorov spectrum" at high wave numbers, suggesting indeed that this breakdown has occurred. The present work aims at extending Mansour and Wray's work to grid turbulence. We will investigate how the Kolmogorov-scaled power spectrum evolves as the Reynolds number continues to decrease, with the view of determining the critical Reynolds number below which the Kolmogorov scaling breaks down. A second aim is to determine whether local isotropy is still valid and how the structure functions behave when the Kolmogorov scaling has broken down.
The direct numerical simulation (DNS) is based on the lattice Boltzmann method (LBM). Rather than solving the governing fluid equations (Navier-Stokes equations), the LBM solves the Boltzmann equation on a lattice [3]. The method was successfully used to simulate turbulent flows [*e.g.* 4, 5, 6]. The computational uniform Cartesian mesh consists of 1600 x 240 x 240 mesh points with $\Delta x = \Delta y = \Delta z$ ($x$ is the longitudinal direction and $y$ and $z$ the lateral directions). The turbulence-generating grid (placed at the $x$-node of 180) is made up of 6 by 6 floating flat square elements in an aligned arrangement (see [4]). Each element is represented by 1 x 20 x 20 mesh points and the mesh spacing ($M$) between the centre of two elements is 40 mesh points (i.e. $2D$), yielding a grid solidity of 0.25. The downstream distance extends to $x/D = 70$ (equivalently $x/M = 35$), where the origin of $x$ is the grid plane and $D = 20$ mesh points is the block side length. Periodic conditions are applied in the $y$ and $z$ directions. At the inlet, a uniform velocity ($U_0 = 0.05$, and $V_0 = W_0 = 0$) is imposed, and a convective boundary condition is applied at the outlet. A no-slip condition at the grid elements is implemented with a bounce-back scheme [7]. The Reynolds number, $R_M$, is varied between 1600 and 3200.

## Results and Discussion

Figure 1 compares the 1D and 3D spectra for the present simulation with those of existing DNS [1, 8] and measurements [9]. The present 3D spectra follow reasonably well those of Mansour and Wray [1] at similar Reynolds numbers. There is a clear deviation from the spectrum of Comte-Bellot and Corrsin [9] which was measured at $R_\lambda$ = 60.7. The same deviation is also observed in the 1D spectra between the present ones and those of Comte-Bellot and Corrsin [9] ($R_\lambda$ = 60.7) and Abe et al. [8] at $R_\lambda$ = 66 obtained at the centerline of a turbulent channel flow. Notice the flattening of the low Reynolds spectra in both the present and those of Mansour and Wray relative to those of Comte-bellot and Corrsin [9] and Abe et al. [8]. Interestingly, Mansour and Wray [1] observed that the nonlinear terms remain active at low $R_\lambda$, despite the clear absence of an inertial range in the spectra. It should be recalled that Kerr [10] has observed a similar spectral deviation and flattening when $R_\lambda$ = 18.4 in his DNS of 3D periodic box turbulence. Figure 1 indicates that the deviation increases as $R_\lambda$ decreases, thus corroborating Mansour and Wray's [1] conclusion that "the shape of the spectra at the Kolmogorov length scales is Reynolds number dependent" at low Reynolds numbers. It thus appears that the Kolmogorov scaling breaks down when $R_\lambda \leq 20$. Note however that the critical value $R_{\lambda,C}$ at which the spectral deviation begins may be higher than 20. Figure 1 suggests that this value is likely to be less than 60; Mansour and Wray [1] proposed a value of 50. Clearly, further measurements and/of DNS are required to determine the actual value of $R_{\lambda,c}$.

We are currently carrying out tests on local isotropy at low Reynolds numbers using second and third-order structure functions. The results will be presented at the conference.

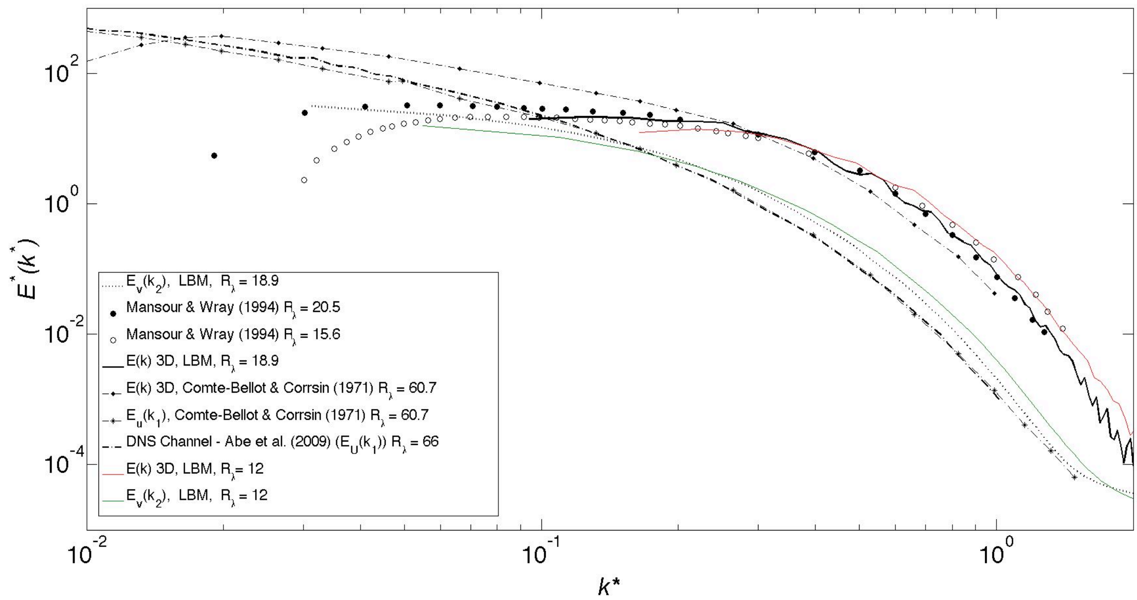


**Figure 1.** 1D and 3D spectra for box and grid turbulence. A spectrum on the centerline of a channel is also shown.

## References


[1] N. N. Mansour and A.A.Wray, 1994 Decay of isotropic turbulence at low Reynolds number, *Phys. Fluids*, **6**, 808-813, 1994.

[2] L. Djenidi and R.A. Antonia, A spectral chart method for estimating the mean turbulent kinetic energy dissipation rate, *Expts. in Fluids*, 2012

[3] U. Frisch, B. Hasslacher and Y. Pomeau, Lattice gas automata for the Navier-Stokes equations, *Phys. Rev. Lett.* 56, 1505-1508, 1986.

[4] L. Djenidi, Lattice Boltzmann simulation of a grid-generated turbulence, *J. Fluid Mech.*, **552**, 13-35, 2006.

[5] P. Burattini, P. Lavoie, A. Agrawal, L. Djenidi, and R.A. Antonia, Power law of decaying homogeneous isotropic turbulence at low $R_\lambda$, *Phy. Rev. E* **73** (066304), 1-7, 2006.

[6] L. Djenidi, Study of the structure of a turbulent crossbar near-wake by means of Lattice Boltzmann. *Phys. Rev. E* **77** (036310), 2008

[7] S. Succi, The lattice Boltzmann equation for fluid dynamics and Beyond, Numerical Mathematics and Scientific Computation, Oxford University Press, 2001.

[8] H. Abe, R. A. Antonia and H. Kawamura, Correlation between small-scale velocity and scalar fluctuations in a turbulent channel flow, *J. Fluid Mech.,* **627**, 1–32, 2009.

[9] G. Comte-Bellot and S. Corrsin, Simple Eulerian time correlation of full- and narrowband velocity signals in grid generated, 'isotropic' turbulence. *J. Fluid Mech.* **48**, 273-337, 1971.

[10] R. Kerr, Higher-order derivative correlations and the alignment of small scale structures in isotropic numerical turbulence, *J. Fluid Mech.,* **153**, 31–58, 1985.