FLOW TOPOLOGY IN DRIFT-WAVE TURBULENCE

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Abstract A study of the relationship between Lagrangian statistics and flow topology in drift-wave turbulence is presented. The topology is characterized using the Okubo-Weiss criterion, which provides a conceptually simple tool to partition the flow into topologically different regions. The turbulent flow considered is governed by the Hasegawa-Wakatani description of drift-wave turbulence. This description has the particularity that it allows to study both Charney-Hasegawa-Mima dynamics and two-dimensional Navier-Stokes turbulence using the same equations by varying a control parameter which is called the adiabaticity. In different flow regimes, the probability density functions of residence time in the topologically different regions are computed using the Lagrangian Weiss field, i.e., the Weiss field along the particles trajectories. In elliptic and hyperbolic regions, the pdfs of the residence time have self-similar algebraic decaying tails. In contrast, in the intermediate regions the pdf has exponential decaying tails.

THE HASEGAWA-WAKATANI DESCRIPTION OF DRIFT-WAVE TURBULENCE

The model assumptions introduced by Hasegawa and Wakatani [1] lead to a model, which describes the dynamics of turbulent fluctuations at the edge of plasma devices such as tokamaks. The resulting equations can be interpreted as a set of equations describing the dynamics of the vorticity, ω and of an advected scalar field, representing plasma density fluctuations. The electrostatic potential, ϕ, plays the role of a streamfunction, such that the velocities in the x and y direction are given by $\boldsymbol{u} = \nabla_\perp \phi$, *i.e.*, $u_x = -\partial\phi/\partial y$ and $u_y = \partial\phi/\partial x$.

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\omega = [\omega, \phi] + c(\phi - n), \tag{1}$$

$$\left(\frac{\partial}{\partial t} - D\nabla^2\right)n = [n, \phi] - \boldsymbol{u}\cdot\nabla\ln(\langle n\rangle) + c(\phi - n), \tag{2}$$

in which all quantities are suitably normalized. Nonlinearities are written as Poisson brackets $[a, b] = \frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x}$. The source term in eq. (2) is the term involving the mean plasma density profile $\langle n\rangle$. The adiabaticity c is a function of different plasma parameters. In our investigation it will be used as a parameter which allows to control the flow regime. Equations (1,2) were solved in a double-periodic domain of size 64^2 using a fully dealiased pseudo-spectral method at a resolution of 1024^2 gridpoints [2]. The adiabaticity is varied between $c = 0.01$ and $c = 2$, to obtain different flow regimes.

FLOW TOPOLOGY

The characterization of the flow topology used here is based on the Okubo-Weiss criterion [3, 4] which is an approximate method of partitioning a flow field into topologically distinct regions. Figure 1 shows snapshots of the vorticity fields for hydrodynamic and adiabatic regimes. From the physics point of view the flow is divided into vorticity dominated regions, which correspond to elliptic regions, strong deformation regions which correspond to hyperbolic regions and intermediate regions. The partition is based on the relative value of $Q = s^2 - \omega^2$ where $\omega = \partial_x v - \partial_y u$ is the vorticity and $s^2 = s_1^2 + s_2^2$ is the deformation with $s_1 = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$, $s_2 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$.
Using the local value of Q, the flow domain can be partitioned in three disjoint regions: i) strongly elliptic for which $Q \le -Q_0$, ii) strongly hyperbolic for which $Q \ge Q_0$ and iii) intermediate regions for which $-Q_0 < Q < Q_0$; where Q_0 is arbitrarily chosen as the standard deviation of the values of Q, *i.e.*, $Q_0 = \sqrt{\langle Q^2\rangle}$ where $\langle\cdot\rangle$ is the ensemble average.

RESIDENCE TIME AND CONDITIONAL LAGRANGIAN STATISTICS

The residence time is the time that a particle stays in the same region of the partition. The pdfs of the residence time conditioned with respect to the three-level Lagrangian Weiss value are shown in Fig. 2. The main behavior, which was observed in [5], is still present for regimes obtained with the Hasegawa-Wakatani model: algebraical tails for strong hyperbolic and elliptic regions and exponential decay for intermediate regions. The influence of adiabaticity is weak for strong elliptic and hyperbolic regions, the exponents do not differ a lot. For intermediate regions, the adiabaticity has

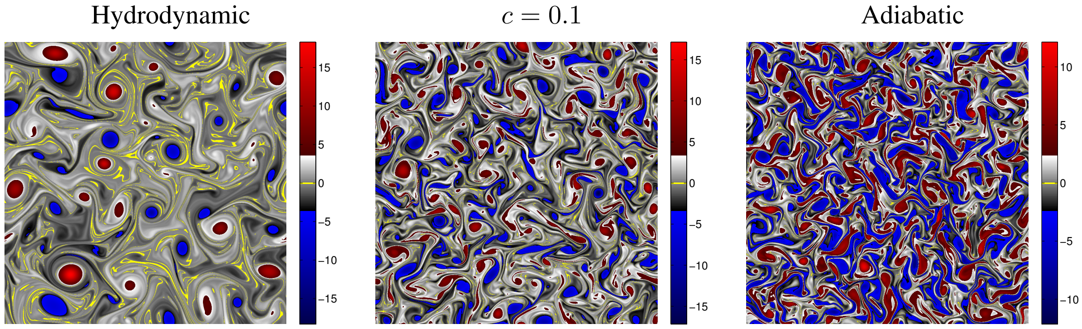

Figure 1. Snapshots of the vorticity field. For low adiabaticity $c \leq 0.7$, the regimes are close to the ones obtained in hydrodynamics (left and middle) while for $c = 0.7$ we obtain a regime relevant to plasma in the edge of tokamak devices (right).

a stronger influence by changing significantly the slope between low and large adiabaticites. We can note also that the behavior is the same for large adiabaticity values $c \geq 0.7$.

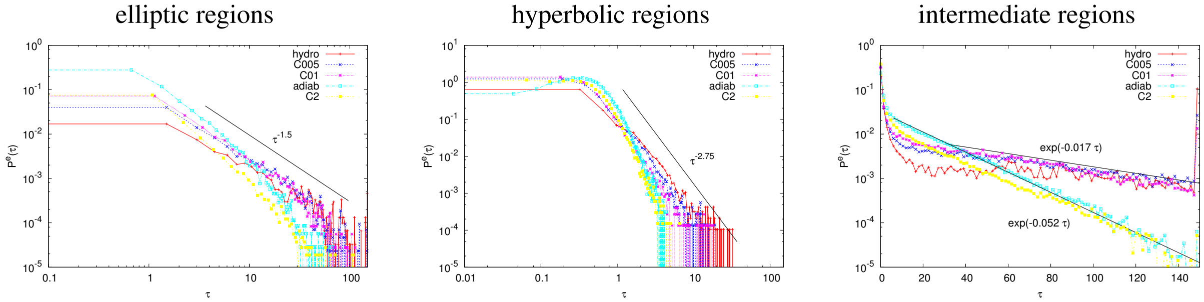

Figure 2. PDFs of residence time conditioned with respect to the three-level Lagrangian Weiss value.

The PDFs of conditional Lagrangian acceleration with respect to three-level Lagrangian Weiss value are shown in Fig. 3. This confirms the results obtained in [2]. We observe an increase of standard deviation for Lagrangian acceleration with decreasing adiabaticity. For large adiabaticity, the Lagrangian acceleration pdfs exhibit a Laplace distribution, while the PDFs of Lagrangian acceleration are close to the pdfs found in hydrodynamic turbulence [5]. The contribution of hyperbolic regions does not seem to change with varying adiabaticity while the one of elliptic regions becomes decreasingly important for increasing adiabaticity. For large adibaticity, both contributions are almost equivalent. Moreover, strong elliptic and hyperbolic regions contribute the most to the Lagrangian acceleration.

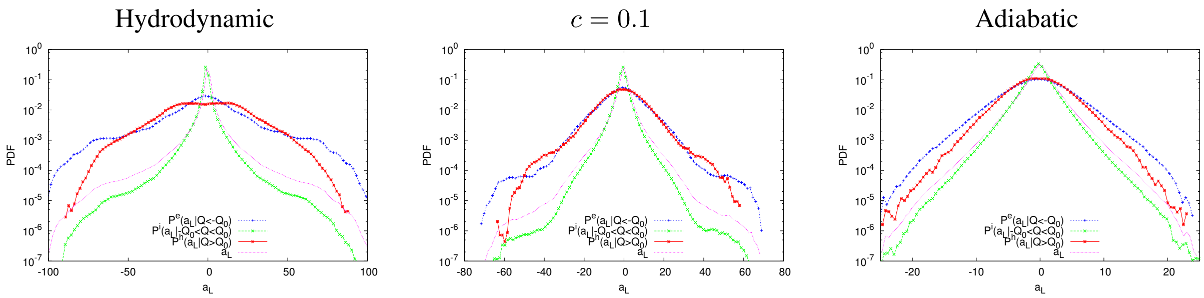

Figure 3. PDFs of conditional acceleration with respect to the three-level Lagrangian Weiss value.

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