ON SCALINGS IN FORCED 2D TURBULENCE

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Abstract We investigate numerically the validity of Kraichnan-Batchelor scalings [1, 7] for forced 2D turbulence. We use different numerical algorithms and different ways of forcing the flow in order to get results as much independent possible of the conditions of experiments. We systematically observe a deviation from the classical $k^{-5/3}$ scaling in the energy inverse cascade inertial range, measuring a steeper slope proportional to k^{-2}. This steepening is related to the emergence of a population of vortices that dominate the distribution of energy across scales, and whose number density and vorticity distribution with respect to vortex area can be related to the shape of the spectrum.

NUMERICAL EXPERIMENTS

To investigate the validity of theoretical scalings in forced 2D turbulence, we proceed to numerical experiments in which both the numerical algorithms and the forcing types are varied. First, we use classical spectral methods with ordinary viscosity, see Scott [8] for details. Second, we use the Combined Lagrangian Advection Method (CLAM), a new hybrid numerical method that combines the efficiency of Lagrangian contour dynamics and vortex methods with the energy conservation properties of the pseudo-spectral method, see Fontane, Dritchel and Scott [5] and references therein for details. For all simulations, no large scale friction nor hypo-diffusion schemes is present in order to avoid any distortion of the inverse cascade and to be also in agreement with the theoretical framework used by Kraichnan. The lack of large-scale dissipation means that our simulations are not stationary in the strict sense, because energy grows continuously, but may be considered quasi-stationary in the sense that the energy distribution in a subrange of the inverse cascade is thus stationary. In fact, it is precisely this quasi-stationary situation that was originally considered by Kraichnan, in which energy cascades undissipated towards ever large scales. We consider narrow band and large band spectral forcing as well as spatial forcing obtained by randomly introducing point vortices (monopoles, dipoles or quadrupoles) in the flow. This original spatial forcing is made possible by use of the CLAM algorithm [3, 6] and has never been tried before since the commonly used spectral methods are not designed for it.

RESULTS

Our results support the recent finding of Scott [8], namely that when a direct enstrophy cascading range is well-represented numerically, a steeper energy spectrum proportional to k^{-2} is obtained in place of the classical $k^{-5/3}$ prediction. With a simple analytical model which closely matches the numerical spectra between the large scales and the forcing scale, we show that this steep spectrum is associated with a faster growth of energy at large scales, scaling like t^{-1} rather than Kraichnan's prediction of $t^{-3/2}$.

The population of coherent vortices that emerge at the forcing scale and grow through multiple interactions is responsible for the deviation of the present results from the theoretical predictions [4, 5]. Indeed, in his model Kraichnan assumed that energy transfers occur locally in spectral space whereas coherent vortices are localised in physical space and therefore are widely distributed in spectral space. This is why we examine the characteristics of the vortex population, in particular their number density and their core vorticity as a function of their size. We consider a decomposition of the full vorticity field into a component associated with coherent vortices and a residual. To define the coherent part we first identify contiguous regions of vorticity whose magnitude is above the rms value, following the method described in Fontane *et al.* [5]. We then consider the shape of each contiguous region by calculating the following parameter

$$\delta_v = \frac{\left(\iint_A \omega dA\right)^2}{2\pi \iint_A \omega^2 \left|\boldsymbol{x} - \boldsymbol{x}_v\right|^2 dA} > 0.5\,, \tag{1}$$

where

$$\boldsymbol{x}_v = \frac{1}{\mathcal{Z}_v} \iint_A \omega^2 \boldsymbol{x}\, dA \quad \text{and} \quad \mathcal{Z}_v = \iint_A \omega^2\, dA \tag{2}$$

are the centroid and enstrophy of the region. Figure 1 illustrates the vortex identification procedure for a representative CLAM simulation with spectral narrow-band forcing.

With this decomposition, enstrophy spectra associated with each field are obtained, and the field of coherent vortices contains most of the energy and enstrophy, except at the very smallest scales. Moreover, the vortex population exhibits

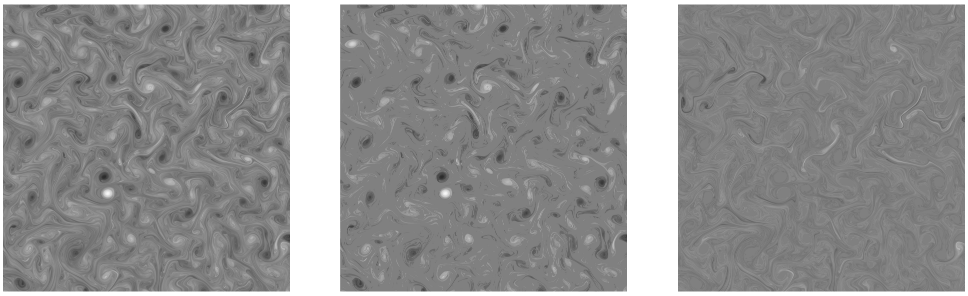

Figure 1. Decomposition of the vorticity field (left) into coherent (middle) and incoherent part (right). The images are screen-shots at time $t = 10$ of one CLAM simulation with narrow band spectral forcing at $k_f = 64$. Only one sixteenth of the domain is shown.

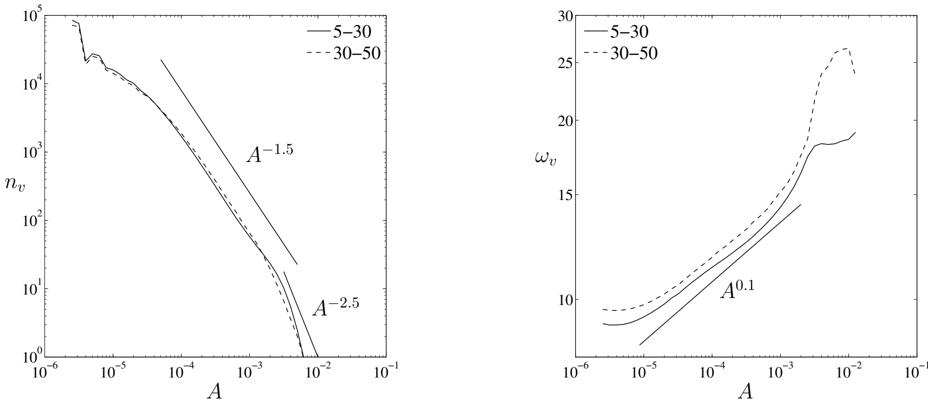

Figure 2. Vortex population characteristics for CLAM simulations with narrow band spectral forcing at $k_f = 64$: vortex number (left) and average vorticity (right) distribution function of vortex area. Data are averaged over two periods: $t \in [5, 30]$ and $t \in [30, 50]$.

a k^{-2} energy spectrum in the inverse cascade range while the background residual follows Kraichnan's prediction with a spectrum $\propto k^{-5/3}$, indicating a structureless field of filamentary debris, consistent with Figure 1. By examining the number density distribution of the vortex population, we can relate more precisely the shape of the spectrum to the vortices. Benzi *et al.* [2] argued that algebraic energy spectra of the form k^{-p} for $p > 3$ may be associated with the emergence of a distribution of vortex sizes, *i.e.* an algebraic number density distribution $n_v(A) = c\, A^{-q}$, provided that the average vorticity ω_v in each vortex does not vary significantly with its area A. We can write the coherent enstrophy as

$$\mathcal{Z}_{\rm coh} = \int Z_{\rm coh}(k)\, dk = \frac{1}{A_s} \int \omega_v^2\, A\, n_v(A)\, dA\ , \tag{3}$$

where A_s is the size of the domain considered. Identifying A with k^{-2}, it follows that $Z_{\rm coh}(k) \propto k^{-5+2q}$ or equivalently $E_{\rm coh}(k) \propto k^{-7+2q}$. Thus the observed k^{-2} spectrum must be related to a vortex number density distribution close to $n_v(A) \propto A^{-2.5}$, as this seems to be the case in Figure 2. Nevertheless, the separation between the forcing scale and the box size is relatively limited and there is a slight growth of the average vorticity with vortex area, see Figure 2. These two elements do not enable us to be definitely conclusive here.

References

[1] G. Batchelor. Computation of the energy spectrum in homogeneous two-dimensional turbulence. *Phys. Fluids*, **12**: II-233–II-239, 1969.

[2] R. Benzi, M. Colella, M. Briscolini, and P. Santangelo. A simple point vortex model for two-dimensional decaying turbulence. *Phys. Fluids A*, **4**: 1036–1039, 1992.

[3] D. Dritschel, R. Scott, C. Macaskill, G. Gottwald, and C. Tran. A unifying scaling theory for vortex dynamics in two-dimensional turbulence. *Phys. Rev. Lett.*, **101**: 094501, 2008.

[4] R. Scott. Nonrobustness of the two-dimensional turbulent inverse cascade. *Phys. Rev. E*, **75**: 046301, 2007.

[5] J. Fontane, D. Dritschel and R. Scott. Vortical control of forced two-dimensional turbulence. *Phys. Fluids*, **25**: 015101, 2013.

[6] J. Fontane and D. Dritschel. The HyperCASL algorithm: A new approach to the numerical simulation of geophysical flows. *J. Comput. Phys.*, **228**: 6411–6425, 2009.

[7] R. Kraichnan. Inertial ranges in two-dimensional turbulence. *Phys. Fluids*, **10**: 1417–1423, 1967.

[8] R. Scott. Nonrobustness of the two-dimensional turbulent inverse cascade. *Phys. Rev. E*, **75**: 046301, 2007.