FRICTION FACTOR FOR TURBULENT FLOWS OF HERCHEL-BULKLEY FLUIDS IN ROUGH PIPES

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Abstract The present work develops a friction factor equation for turbulent flows of Herschel-Bulkley fluids in rough pipes. A new characteristic velocity that takes into account the influence of the yield stress and reduces to the classical friction velocity in the Newtonian fluid limit is proposed. The theoretical predictions are compared with experimental data obtained for four types of different roughnesses, showing good agreement. The influence of the rheological parameters on the friction factor is fully discussed.

INTRODUCTION

The adequate description and prediction of turbulent flows of non-Newtonian fluids in rough pipes, remains one the most challenging topics in the chemical, mechanical and petroleum industries. Even the behavior of purely viscous fluids cannot be said to be well understood. The scarcity of experimental data on rough pipe flows has surely prevented researchers from correctly characterizing the influence of roughness on the structure of non-Newtonian flows.

Most of the work on purely viscous fluids in turbulent pipe flow follows the analysis of the pioneering paper of Dodge e Metzner(1959)(DM), who developed a skin-friction equation for power-law fluids. Despite the popularity of the expression of D&M for the friction factor, it does not take into account the effects of yield stress and wall roughness. In fact, very few works can be found in literature with any profound discussion on the theoretical or experimental aspects of turbulent non-Newtonian flows in rough pipes. Szilas et al. (1981) have proposed a skin-friction but their expression does not incorporate the above effects.

In the present work, a local velocity profile and a friction coefficient equation are deduced for turbulent flow of Herschel-Bulkley in rough pipes. A logarithmic velocity profile is obtained considering a two-layered turbulent configuration, an outer wake-like structure and an inner wall layer. A new characteristic velocity that takes into account the influence of the yield stress and that is different from the classical friction velocity is proposed. The proposed relation reduces to the classical friction-coefficient power-law expression as the yield stress and wall roughness influence tend to zero. The mathematical description of the proposed velocity is shown below:

$$u_c = \sqrt{\frac{\tau_w}{\rho} - \frac{\tau_0}{\rho}} \tag{1}$$

In equation (1), τ_w represents the shear stress at the wall, τ_0 is the yield stress and ρ is the fluid density. It is important to notice that u_c reduces to the classical friction velocity as the yield stress vanishes.

The impact of the fluid rheological parameters and the wall roughness is discussed next. Figures 1 and 2 show the effect of the yield stress, through the generalized Hedstrom number H_{eg}= $D^{2n/(2-n)}(\tau_0/\rho)/(K/\rho)^{2/n-2}$ and the wall roughness (ϵ/D) on f_d (Darcy's friction coefficient) for smooth and rough walls (D is the pipe diameter, K is the consistency index of the fluid, n is the power law exponent). The generalized (Dodge and Metzner (1959)) Reynolds number is defined as Re_{DR} = $8(\rho U^{2-n} D^n)/(K(6+(2/n)))$. The results are presented here for n = 1/2 and n = 1 for various values of the generalized Hedstrom number. Clearly the friction coefficient increases with increasing values of the Hedstrom number, and decreases for decreasing values of the exponent index due to the shear thinning effects. The skin-friction equation is written as follows:

$$\frac{1}{\zeta} = -\frac{\ln 10}{\varkappa} \log \left(\frac{2}{R_{e_{DR}}^{1/n} \frac{6+2n}{8^{1/n}} \left(\frac{\zeta}{\sqrt{8}}\right)^{(2-n)/n}} + 0.6 \frac{\epsilon}{D} \right) + B - \frac{3}{2\varkappa} \tag{2}$$

with

$$\zeta = \sqrt{f_d - 8 \frac{H_{eg}}{R_{e_{DR}}^{2/(2-n)}}} \tag{3}$$

$$B = -\frac{0.55}{n^{1.2}} + \frac{2.5}{n^{0.75}} \left(1.96 + 0.816n - 1.628 \log\left(3 + n^{-1}\right)\right) \tag{4}$$

$$\varkappa = 0.41 n^{-0.25} \tag{5}$$

where ϵ is the roughness characteristic length.

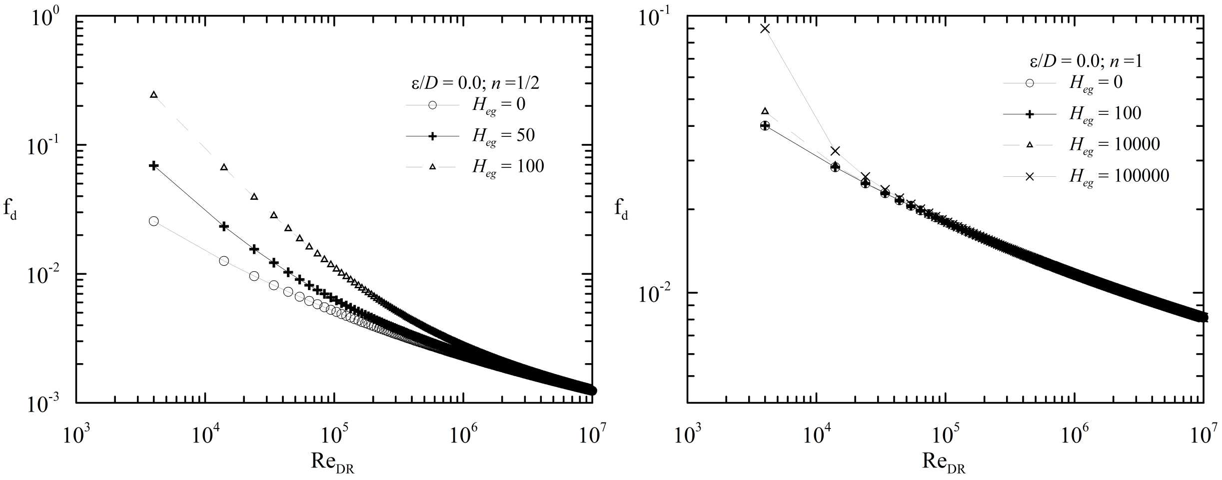

Figure 1. Friction factor for n = 1/2 and 1 respectively, ϵ/D = 0.0 (smooth wall).

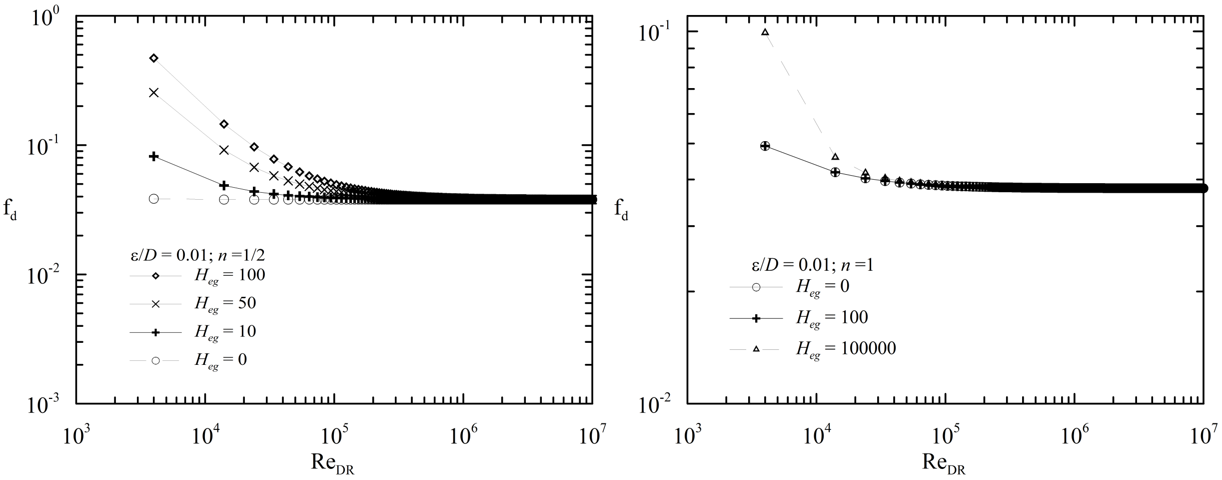

Figure 2. Friction factor for n = 1/2 and 1 respectively, ϵ/D = 0.01.

Equation (2) was derived through the logarithmic velocity profile of the turbulent region, and reduces to the friction equation introduced by Dodge and Metzner (1959), as corrected by Skelland (1967), for τ_0 = 0 and ϵ = 0. According to Eq. (2) the friction factor tends to a constant value for large values of $R_{e_{DM}}$, characterizing the fully rough regime (Figure 2). It is important to notice that the fully rough asymptotic value is not the same for different values of the exponent index n. In fact, it decreases with decreasing values of n. That behavior is a consequence of the influence of the exponent index n on B. The influence of H_{eg} on f_d is pronounced for shear thinning fluids (n = 1/2, Figures 1 and 2).

CONCLUSION

In this work, a friction equation that considers the influence of the wall roughness on turbulent flows of Herschel-Bulkley fluids is proposed. The equation is obtained through some asymptotic arguments and a matching procedure to yield a logarithmic velocity profile in the fully turbulent region. A new characteristic velocity that accounts for the influence of the yield stress and includes in its definition the classical friction velocity as a particular case is proposed from dimensional arguments.

References

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