THE GEOMETRY OF THE TURBULENT-NONTURBULENT INTERFACE LAYER IN BOUNDARY LAYERS

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Abstract
The method to obtain the turbulent-nonturbulent interface by defining a threshold for a variable that depends on turbulent fluctuations is revisited. Instead of choosing the threshold from practical considerations, it is considered a parameter. The goal is to find if a particular value for the threshold emerges from the geometrical properties of the resulting surface and the characteristics of the flow at its vicinity. For this purpose, only methods that are consistent with a three-dimensional fractal-like nature of the surface are applied. A convention for the threshold, that depends only on the geometry of the interface, applicable to different flows, is proposed. However, when this criterion is applied to different scalar fields, or alternative definitions of distance are applied, different results are obtained. This has a direct consequence on any quantity that is obtained from the surface's geometry, such as normals and thicknesses.

INTRODUCTION

The usual criterion to define a surface that separates turbulent from irrotational flow is to threshold for a quantity that depends on turbulent fluctuations, like enstrophy [1] and [2] or some vorticity component [3]. When this method is applied, two problems arise. First, figure 1a shows that there is a range of values, with a width of almost one order of magnitude, that could arguabily chosen for a threshold. Second, independently from the choice, one obtains a very complex surface, as can be seen in figures 1b and 1c.
The problem to be solved is therefore to find a non arbitrary value for the threshold, or a method to find it.

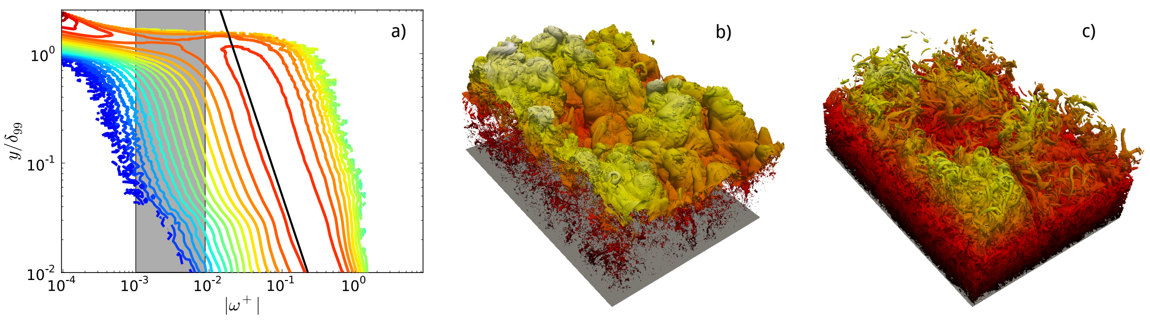

Figure 1. a) Contours for PDF of the logarithm of vorticity magnitude for different wall-parallel planes. A clear difference between the vorticity of the turbulent flow and the residual vorticity of the irrotational flow can be seen. The solid black line is the expected scaling from the wall, $|\omega^+| \propto y^{-1/2}$. The shaded area corresponds to a range of possible vorticity magnitudes that could be chosen as a threshold. b) and c) Contour in an instantaneous enstrophy field that corresponds to the lower and higher limit in a).

RESULTS

One turbulent boundary layer in the range $Re_\theta = 2400 - 6800$ ($Re_\tau = 1000 - 2000$) with natural entrainment rate [4], and a second one at $Re_\theta = 1300 - 4600$ ($Re_\tau = 600 - 1500$), where the entrainment rate has been incremented by 75% [5], have been be used in this analysis.
The properties of the surface obtained will be analyzed as functions of the threshold. The intention is to find if a particular value, or a small set of them, can be used to locate the turbulent-nonturbulent interface. The approach has been to focus on the topological properties of the largest connected surface generated by thresholding the vorticity magnitude. Special attention is paid to use only methods that can be applied to fractal surfaces. In consequence, no spatial direction will be privileged over the others and no spatial derivatives, which depend on the scale considered, will be computed. Vorticity magnitude has been chosen over ω_z because the same process generates a much more complex object for the latter, as it can be seen in figure 2a.
Figures 1b and 1c show that the shape of the surface is very different at the lower and the higher limits of the possible threshold. This suggests that the geometrical properties of the surface could be used to define a characteristic threshold. Three quantities are analyzed: the number of connected objects, the genus of the largest surface (equivalent to the number of handles), and its fractal dimension. Their values are shown in figures 2b and 2c depending on the threshold. With a very low threshold, the whole conditioned scalar field is a single connected object with almost no handles in it, and

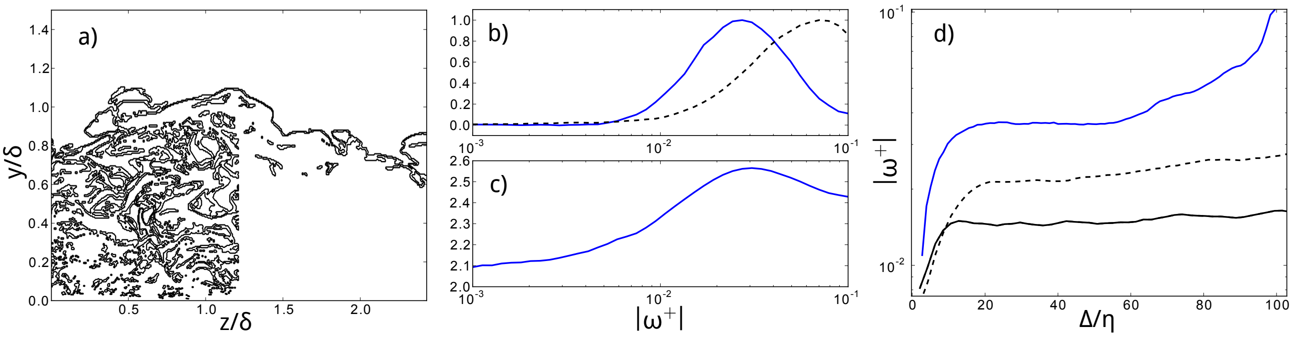

Figure 2. (a) Cross stream cut of the points contained in the largest surface detected for a threshold of $\omega_z = |\omega|/\sqrt{3}$ (left) and vorticity magnitude (right). (b) Evolution of the genus (solid), and number of objects (dashed) normalized with their maximum value; depending on the threshold for a vorticity magnitude field. (c) Evolution of the lower Kolmogorov capacity. (d) Average value of the thresholded field for different definitions of distance: minimum-ball distance in a vorticity magnitude field (upper blue), vertical distance in a vorticity magnitude field (dashed black), and vertical distance in an ω_z field (lower black). Distances are normalized with the Kolmogorov viscous scale at $y/\delta = 0.6$

has a low fractal dimension. As the threshold increases, there is still a single object where handles start to appear; while the fractal dimension increases smoothly. The genus could be used as a criterion to fix a characteristic value, because it changes from practically zero to tens of thousands for a domain of size $2.5\delta \times 2.5\delta$ at $Re_\tau = 1500$, and the shape of the evolution does not depend on the Reynolds number. For the following analysis the threshold will be fixed when the genus is 5% the maximum.

The surface that is subject of analysis is a fractal within a range of scales, as can be seen in figure 2c. This should be taken into account when quantities that depend on its geometrical properties, like thicknesses, normals, or areas; are computed. It is also important when some property of the flow is related to the position relative to the surface, because one needs a definition of distance.

The following method is proposed. Instead of choosing a privileged direction along which the distance is computed, like [1], [2] and [3]; the minimum-ball distance is used. This definition is valid for a fractal, and takes into account that the surface can be folded and can contain handles. To compute the closest point of the surface to a given position a fast neighbour search is required; in this case a k-d tree is used [6]. This allows us to add another scalar field to the flow: the distance to the closest point in the surface. The next step is to compute the joint PDF of any scalar quantity, vorticity magnitude in this case, and the distance for any point in the flow.

For instance, the mean vorticity magnitude found to a given distance to the surface is shown in figure 2d. It has been obtained conditioning the joint PDF of the values of vorticity magnitude higher than the threshold. This result is compared with other conditioned vorticity profiles found in the literature: vertical distance between the first detection of the surface for a vorticity magnitude field [1],[2], and vertical distance for a ω_z field. The conditional profiles are very sensitive to the definition of distance and to the variable used, suggesting that values given for the turbulent-nonturbulent interface thickness depend on the method used to compute them.

ACKNOWLEDGEMENTS

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