The Lack of Return to Isotropy in Decaying, Axisymmetric, Saffman Turbulence

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We consider freely-decaying, anisotropic, statistically-axisymmetric Saffman turbulence in which $E(k \to 0) \sim k^2$. As noted in Saffman (1967a) and Krogstad & Davidson (2010), this represents a good model of certain classes of grid turbulence in a wind-tunnel. We note that such turbulence possesses two statistical invariants which are related to the form of the spectral tensor $\Phi_{ij}(\mathbf{k})$ at small *k*. These are (Davidson, Okamoto & Kaneda, 2012)

$$M_{//} = \Phi_{//}(k_z = 0, k_\perp \to 0) = (2\pi)^{-3} \lim_{k_\perp \to 0} \int e^{-j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \langle u_{//} u'_{//} \rangle d\mathbf{r} = \text{constant},$$

and

$$\tfrac{1}{2} M_\perp = \Phi_\perp(k_z = 0, k_\perp \to 0) = (2\pi)^{-3} \lim_{k_\perp \to 0} \int e^{-j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \langle \mathbf{u}_\perp \cdot \mathbf{u}'_\perp \rangle d\mathbf{r} = \text{constant},$$

where $\langle u_i u'_j \rangle$ is the usual two-point velocity correlation, the subscripts // and $\perp$ indicate quantities parallel and perpendicular to the axis of symmetry and $\Phi_{//} = \Phi_{zz}, \Phi_\perp = \Phi_{xx} + \Phi_{yy}$. Since $M_{//} \sim u_{//}^2 \ell_\perp^2 \ell_{//}$ and $M_\perp \sim u_\perp^2 \ell_\perp^2 \ell_{//}$, self-similarity of the large scales (when it applies) demands that $u_{//}^2 \ell_\perp^2 \ell_{//} = \text{constant}$ and $u_\perp^2 \ell_\perp^2 \ell_{//} = \text{constant}$. This, in turn, requires that $u_{//}^2 / u_\perp^2$ is constant, contrary to the popular believe that freely-decaying turbulence should exhibit a 'return to isotropy'. Numerical simulations performed in large periodic domains, with different types and levels of initial anisotropy, confirm that $M_{//}$ and $M_\perp$ are indeed invariants and that, in the fully-developed state,

$$u_{//}^2 \ell_\perp^2 \ell_{//} = \text{constant}, \quad u_\perp^2 \ell_\perp^2 \ell_{//} = \text{constant},$$

from which

$$u_{//}^2 / u_\perp^2 = \text{constant}, \quad \text{(fully-developed turbulence).}$$

Somewhat surprisingly, the same simulations also show that $\ell_{//} / \ell_\perp$ is more or less constant in the fully-developed state (figure 1). Simple theoretical arguments then suggest that, when $u_{//}^2 / u_\perp^2$ and $\ell_{//} / \ell_\perp$ are both constant, the integral scales should evolve as $u_\perp^2 \sim u_{//}^2 \propto t^{-6/5}$ and $\ell_\perp \sim \ell_{//} \propto t^{2/5}$, irrespective of the level of anisotropy and of the presence of helicity. These decay laws, first proposed by Saffman (1967b), are verified by the simulations.

References

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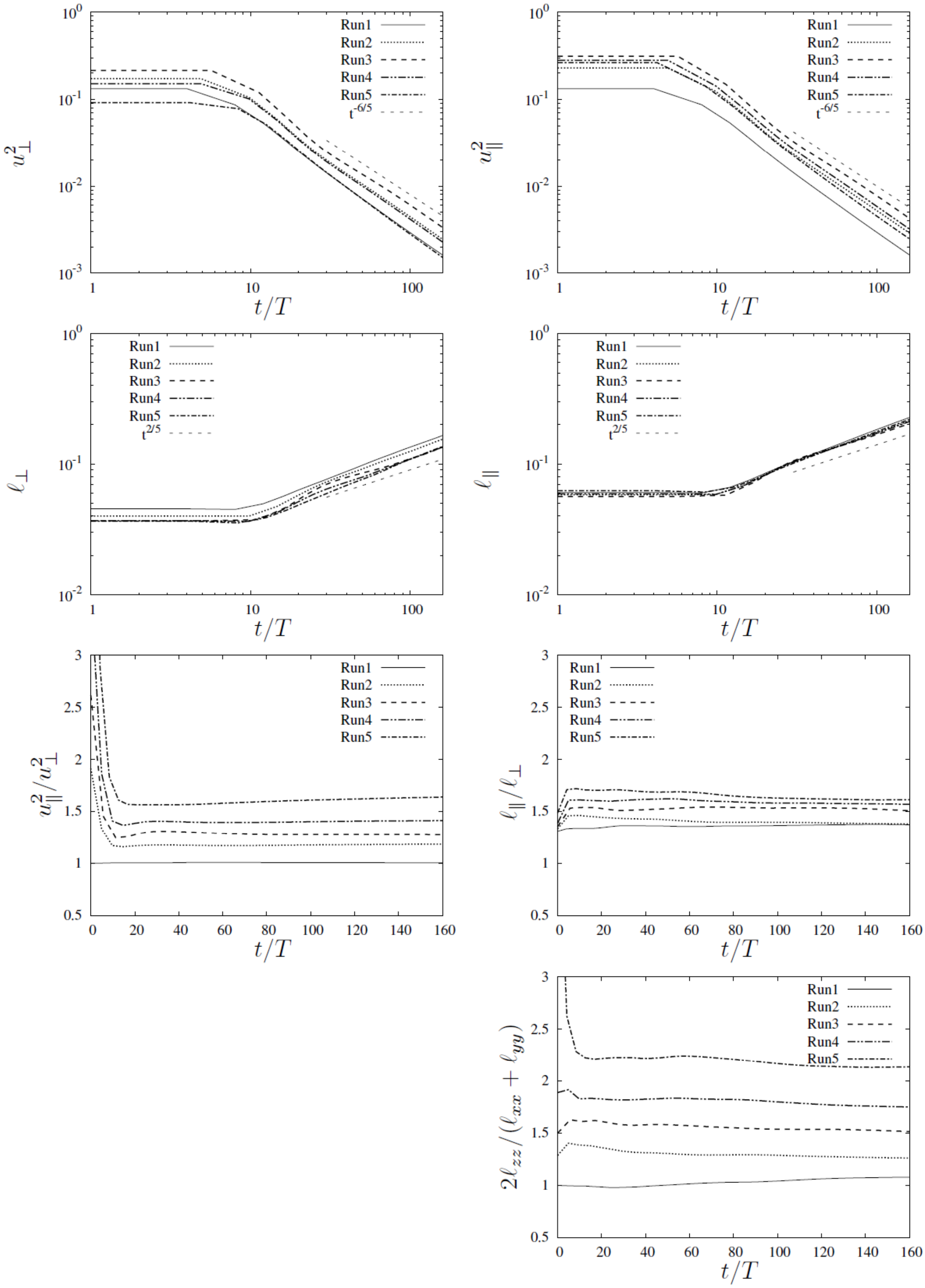

Figure 1. The variation of energy, integral length scales and anisotropy in 5 runs with different types and levels of initial anisotropy.