RELATIVE PERIODIC EDGE ORBITS IN PLANE CHANNEL FLOW

Subhandu Rawat[1], Carlo Cossu[1] & Francois Rincon[2]
[1] *CNRS and Université de Toulouse, IMFT; Allée du Pr. Camille Soula, 31400 Toulouse, France*
[2] *CNRS and Université de Toulouse, IRAP ; 14 avenue Edouard Belin, 31400 Toulouse, France*

Abstract A branch of genuine relative periodic orbits is found to be an edge state in plane Poiseuille flow in a periodic domain. These periodic solutions correspond to sinuous quasi-streamwise streaks periodically forced by sinuous quasi-streamwise vortices in a self-sustained process. The rms-amplitude of the streaks is found to scale as $\approx \mathrm{Re}^{-0.8}$, while that of the quasi-streamwise vortices scales like $\approx \mathrm{Re}^{-1.6}$.

MOTIVATION

The 'edge of chaos separates regions in phase space where initial conditions rapidly relax to the laminar state from regions where initial conditions either relax after a turbulent transient or remain turbulent [4]. Edge states are relative attractors within the edge of chaos. In plane Couette flow in constrained periodic domains 'lower branch' nonlinear traveling waves (TW) have been found to be edge states [7]. Chaotic dynamics within the edge has been found in Hagen-Poiseuille flow in a short periodic circular pipe [3]. In the case of plane Poiseuille flow at Re$= 3000$ in a periodic domain of streamwise and spanwise size $L_x = \pi h$ and $L_z = 0.4\pi h$ a TW edge state has been found [2] using a bisection procedure that started by an initial guess based on a turbulent solution. A later study in the same domain a the same Re but with different initial conditions in the bisection procedure, revealed the existence of a "periodic-like solution" [5], i.e. a solution that could be either a genuine periodic solution or an heteroclinic connection (a single period of the solution was accessible due to numerical precision problems). Both solutions were localized on a single wall of the channel. The scope of the present study is to ascertain if genuine periodic edge states exist in plane Poiseuille flow. If found, it is hoped that these states could be continued to 'upper branch' periodic orbits and help in understanding the structure of the turbulent state.

FORMULATION

We consider the flow in a plane channel in the domain $L_x = 2\pi h$ and $L_z = 2.416h$ (h being the channel half-width). This domain is the one in which TW solutions appear at the lowest Reynolds number [8]. The edge is explored using a standard bisection procedure on the initial conditions: $\mathbf{u}_0 = \{U_{lam}(y), 0, 0\} + A_1 \{0, -\partial\psi_0(y,z)/\partial z, \partial\psi_0(y,z)/\partial y\} + A_2 \{0, 0, w_{sin}(x,y)\}$. The stream-function $\psi_0(y,z) = \left(1-y^2\right) \sin\left(2\pi y/L_y\right) \sin\left(2\pi z/L_z\right)$ is associated to streamwise uniform vortices, and $w_{sin}(x,y) = \left(1-y^2\right) \sin\left(2\pi x/L_x\right)$ is a sinuous perturbation. $\mathbf{u}_0$ is solenoidal has the same volume flux of the laminar Poiseuille solution $U_{lam} = 1 - y^2$. The bisection is operated on A_1 with the ratio $A_2/A_1 = 1/10$ kept fixed. The Navier-Stokes equations are integrated using the Fourier-Chebyshev-Fourier pseudospectral `channelflow` code [1] with 16x41x16 points in the streamwise, wall-normal and spanwise directions and enforcing a constant volume flux during the simulation. We have verified that the characteristics of the edge states found by bisection on the coarse grid do not change when the number of collocation points is increased to 32x65x32.

RESULTS

At Re=3000 the edge trajectories converge to a relative periodic orbit (RPO) of period $T = 739$ (convective time units). Similar RPOs are found for Re=4000 and 5000, with respective periods $T = 1090$ and 1418. The projection of the RPOs in the plane $I/I_{lam} - D/D_{lam}$ of the perturbation energy injection and dissipation normalized to the laminar solution values is reported in the right panel of fig. 1. The amplitude of the RPOs is relatively small and decreases when Re is increased which is characteristic of 'lower branch' solutions. This is confirmed by the analysis of the rms velocity perturbations. Most of the perturbation rms kinetic energy resides in the streamwise velocity component (streaks) whose amplitude decreases like $\mathrm{Re}^{-\gamma}$ with $\gamma \approx 0.8$. The rescaled $\mathrm{Re}^{\gamma} u'_{rms}(y)$ profiles, shown in the left panel of fig. 2 are almost identical for the three considered Re and indicate the presence of streamwise streaks with maximum amplitude located near $y \approx \pm h/2$. The wall-normal and spanwise velocity rms profiles are consistent with center-channel quasi-streamwise vortices. The rms amplitude of the wall-normal velocity is proportional to $\mathrm{Re}^{-2\gamma}$, while that of the spanwise velocity is proportional to $\mathrm{Re}^{-\frac{3}{2}\gamma}$. Contrary to previously found TW [8] and periodic-like solutions [5] which were localized near the wall, the velocity fields associated to the found solutions are more reminiscent of detached eddies structures [6].

Acknowledgements: The use of the `channelflow.org` code [1] and financial support from PRES Universté de Toulouse and Région Midi-Pyrénées are kindly acknowledged.

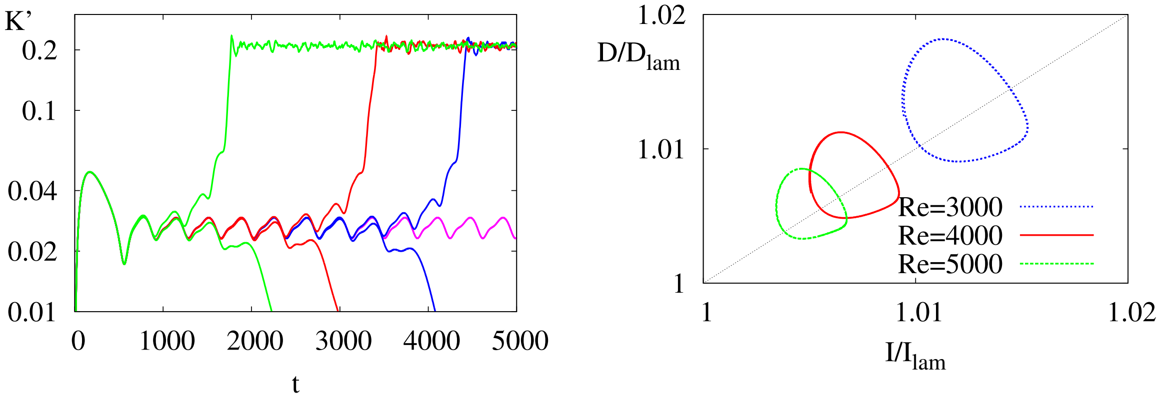

Figure 1. Left panel: Perturbation kinetic energy history for selected iterations of the bisection (Re= 3000). The trajectory on the edge is rapidly attracted to a periodic solution. Right panel: Projection on the normalized $I - D$ plane of the periodic edge states found at Re=3000, 4000 and 5000. The energy injection and dissipation are normalized to the value that the laminar Poiseuille solution would have at the same Re.

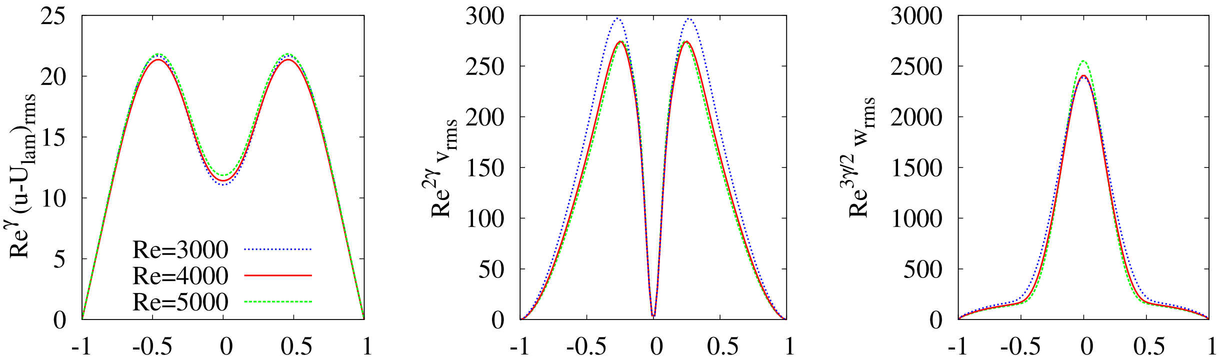

Figure 2. Rescaled root-mean-square amplitudes of velocity perturbations, averaged on a period of the cycle. The streamwise, wall-normal and spanwise components are respectively reported in the the left, center and right panel.

References

[1] J. F. Gibson, J. Halcrow, and P. Cvitanovic. Visualizing the geometry of state space in plane Couette flow. *J. Fluid Mech.*, **611**:107–130, 2008.
[2] T. Itano and S. Toh. The dynamics of bursting process in wall turbulence. *J. Phys. Soc. Jpn.*, **70**:703–716, 2001.
[3] T. M. Schneider, B. Eckhardt, and J. Vollmer. Statistical analysis of coherent structures in transitional pipe flow. *Phys. Rev. E*, **75**:066313, 2007.
[4] J.D. Skufca, J.A. Yorke, and B. Eckhardt. Edge of Chaos in a Parallel Shear Flow. *Phys. Rev. Lett.*, **96**(17):174101, 2006.
[5] S. Toh and T. Itano. A periodic-like solution in channel flow. *J. Fluid Mech.*, **481**:67–76, 2003.
[6] A. Townsend. *The structure of turbulent shear flow*. Cambridge U. Press, Cambridge, UK, 1976. second edition.
[7] D. Viswanath. The dynamics of transition to turbulence in plane couette flow. *ArXiv physics/0701337*, 2007.
[8] F. Waleffe. Homotopy of exact coherent structures in plane shear flows. *Phys. Fluids*, **15**:1517–1534, 2003.