EXPERIMENTS ON THE ONSET OF SUBCRITICAL TURBULENCE IN SHEAR FLOWS

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Abstract The onset of sustained turbulence in pipe flow is a non-equilibrium phase transition between decaying and spreading turbulence [2]. However, the extremely long timescales of these processes in pipe flow make it very difficult to measure important signatures of criticality, such as the scaling of turbulent fraction. This information is necessary to determine the phase transition class of the onset of turbulence in shear flows. With this goal in mind we have built a Taylor-Couette experiment with an aspect ratio of more than 200 and an azimuthal length of more than 300 gap-widths, allowing us to measure turbulent fractions in the vicinity of the critical point.

PREVIOUS WORK IN PIPE FLOW

Pipe flow is a classical example of a shear flow that is linearly stable. Reynolds observed already more than 125 years ago that finite amplitude perturbations are necessary to trigger turbulence [13], which is characteristic for a subcritical transition. At low Reynolds number Re the turbulence appears localized (called puffs in pipe flow) in the otherwise laminar flow and is transient [9, 3, 10]. However, the ability of puffs to spread and split thereby generating another puff can statistically outweigh the decay of individual puffs, leading to a sustainment of turbulence [2, 4]. This non-equilibrium phase transition between transient and sustained turbulence takes place at 2040$\pm$10 [2] and is believed to be of second order [4].

MOTIVATION

In this talk we will investigate how the recent findings in pipe flow apply to other linearly stable shear flows. One of the main differences between pipe flow and for example plane Couette flow, the flow between two shearing plates, is that the turbulence cannot only spread in the stream-wise but also in the span-wise direction. How this additional degree of freedom influences the phase transition is not yet understood. Previous investigations in plane Couette flow measured the mean turbulent fraction of the flow depending on the Re and concluded that this transition is, in contrast to pipe flow, of first order [6, 8]. However, it is not clear that the system size in the experiments and observation times in the numerical simulations was sufficient to unambiguously determine the nature of this transition. Unfortunately large plane Couette experiments are technically very difficult to control [6], and numerical simulations that would match the time scales used in the experiments are currently too expensive to be practical [8]. Another possibility to still study this transition is to use a Taylor–Couette experiment, which is easier to control (including the boundary condition) and offers all the advantages of a closed system. In the TC system the azimuthal length L (at midgap) depends on the radius ratio η of the two cylinders ($L = \pi(1+\frac{1}{\eta})/(\frac{1}{\eta}-1)$): the larger the radius ratio the larger the azimuthal length. We have designed and build a new high-radius ratio Taylor–Couette device to study this transition.

TAYLOR–COUETTE EXPERIMENT

The Taylor–Couette device consists of two concentric rotating cylinder with a radius ratio $\eta = 0.98$. The outer cylinder is made of Borosilicate precision glass with a radius $r_o = 112.54$mm, the inner cylinder is made of polished stainless steel with an radius of $r_i = 110.25$mm, the gap in between has a width of $d = 2.29 \pm 0.075$mm. The azimuthal length is $306d$ (stream-wise direction), the aspect ratio of the gap between the cylinders is $\Gamma = 262d$ (span-wise direction). The top and bottom endplate confining the flow in the axial direction can be rotated independently to minimize finite-size effects. The Reynolds number for the inner and outer cylinder are defined as $Re_{i,o} = \frac{2\pi f_i r_i d}{\nu}$ with the rotation frequency $f_{i,o}$ of the cylinders and the viscosity $\nu = 4.8$ of the working fluid (silicone oil).

The outer cylinder is placed in a squared acrylic box filled with temperature-controlled silicone oil to produce temperature stability and optimize for optical measurements by index matching. The working fluid is seeded with aluminum tracers (less than 1% in volume) for flow visualization. The flow is monitored with a high-speed camera for further analysis, typically sampling with 100Hz.

RESULTS

In the first step of our investigation we map out the primary instability for counter-rotating cylinders with a radius ratio of $\eta = 0.98$. In the second step we focus on the subcritical transition to determine the order of the phase transition and possible belonging to universality classes like directed percolation [7].

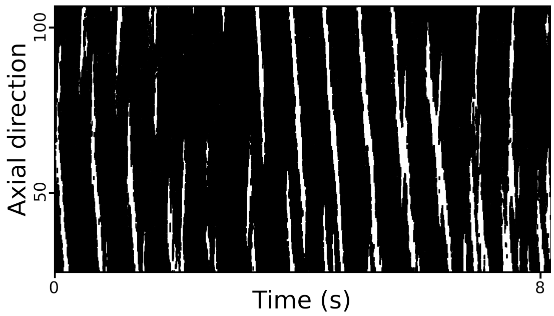

Figure 1. *Left:* Picture of the Taylor–Couette experiment. *Middle:* Snapshot of subcritical turbulence for counter-rotating cylinders. *Right:* Space-time diagram of subcritical turbulence. White regions are turbulent, black indicates laminar flow.

While plane Couette flow is linearly stable for all Re, this is not the case for Taylor–Couette flow. When both cylinders are counter-rotating the flow becomes linearly unstable, which can be predicted by linear stability analysis. We have compared our measurements with the calculated values of this transition for $\eta = 0.98$ and find an excellent agreement (deviation below 1%), which is so far unachieved in high-radius ratio Taylor–Couette experiments.

In order to approximate the subcritical transition border we start from a turbulent state and decrease Re_i for a constant Re_o quasi-statically until the turbulence decays. For $Re_o < 800$ the transition agrees with the linear stability threshold so that no subcritical turbulence exists. But for higher counter-rotation rates a hysteresis appears, qualitatively similar to the phase diagram for $\eta = 0.883$ [1]. But in high radius-ratio Taylor–Couette experiment it is extremely difficult to avoid finite amplitude perturbations triggering turbulence in the subcritical regime. This is the reason why this hysteresis couldn't be observed in earlier investigations [12, 11].

The subcritical turbulence appears in form of spiral turbulence, single spiral arms or spots localized in the steam-wise and span-wise direction (see Figure 1 *Middle*). As Re_i is further decreased the turbulent fraction shrinks until the flow finally relaminarizes. Like in experiments where only the outer cylinder is rotating [5] the decay is a statistical process.

Our analysis of the scaling of the turbulent fraction indicates a continuous (second order) phase transition, in contrast to earlier investigations in Taylor- and plane Couette flows, but in agreement with observations in pipe flow. By an additional analysis of the size distributions of the laminar gaps in the subcritical turbulence it may be possible to clarify if this transition belongs to the directed percolation universality class [7].

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