RANDOM TRANSITIONS IN STOCHASTIC TURBULENT FLOWS

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Abstract Many turbulent systems exhibit random switches between qualitatively different attractors. The transition between two different attractors is often an extremely rare event that can not be computed through direct numerical simulations due to complexity limitations. In this talk, we present results for the calculation of the most probable transition trajectories or *instantons* between non–equilibrium stationary states (attractors) in the 2D stochastic Navier-Stokes equations. By representing the transition probability between two given states as a path integral, we can determine the instanton by the minimization of an appropriate action functional.

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Many non-equilibrium turbulent flows eventually relax to some non-equilibrium statistical steady state. If several such states coexist, then the system may, due to stochastic fluctuations, undergo sporadic random transition one to another quasi-stable flow configuration. This bistable behaviour has been observed in several turbulent systems, including magnetic field reversals of the Earth, or in MHD experiments [1], Rayleigh–Bénard convection cells [5, 8, 6, 4], two–dimensional (2D) turbulence experiments and numerical simulations [12, 9, 2], three–dimensional turbulence flows [10], atmospheric flows [13] and for paths of ocean currents [11].

A straightforward numerical approach in considering these transition events would be to simply perform a direct numerical simulation of the governing equations and to wait until a transition trajectory is observed. Most of the time, this is impracticable due to the extremely long time between subsequent transitions, very high Reynolds number and to the large number of degrees of freedom involved. In this talk, we will consider two–dimensional and geostrophic turbulence models with stochastic forcing. As an alternative to direct numerical simulations , we propose a non–equilibrium statistical mechanics approach to the computation of this phenomenon. Our strategy is based on the large deviation theory for stochastic dynamical systems, namely the Freidlin–Wentzell theory [7], derived from a path integral representation of the stochastic process.

THE LARGE DEVIATION AND INSTANTON APPROACH

The transition probability P is the probability of the system starting from an initial state ω_0, to reach a final state ω_T in a time interval T. It is thus an essential dynamical quantity that encodes most of statistics of the system. For multi-stable systems with weak noise, the transition from one attractor to another is an extremely rare event. However, as paradoxical as it may seem, most of the trajectories connecting two attractors are usually concentrated close to a single one, known as the *instanton* trajectory.

Instanton theory is a way of determining this most probable transition (instanton) trajectory between two states in a non-equilibrium dynamical system with weak noise. The transition probability is represented as a path integral over all possible transition trajectories connecting the two states:

$$P(\omega_0, \omega_T, T) \; = \; \int_{\omega(\mathbf{x},0)=\omega_0}^{\omega(\mathbf{x},T)=\omega_T} \mathcal{D}[\omega] \; \exp\left(-\frac{1}{2\alpha}\mathcal{A}[\omega]\right). \tag{1}$$

Deviation from the deterministic (zero noise) trajectory is represented by a ‘cost’ function that exponentially diminishes the probability the further away from the deterministic trajectory one get. The exponent is proportional to the inverse of the noise amplitude α and the system specific action functional $\mathcal{A}$.

The goal is to compute the that trajectory that contributes the most to the transition probability P. This can be achieved by utilizing the saddle–point approximation, in the weak noise limit ($\alpha \to 0$) to the path integral (1). If the set of transition trajectories are concentrated around a specific one, then the saddle–point approximation is valid and leads to the definition of the instanton being the trajectory that globally minimizes the action functional $\mathcal{A}$ subject to the boundary conditions. The transition probability can then be computed and used to estimate the timescale of observing such a trajectory. Because of the large separation of timescales, usually it is far more efficient to compute the minimizer of the action than to perform a direct numerical simulation of the system until a transition trajectory is observed.

In this talk, we present results on applying the large deviation and instanton approach to bistability in the stochastically forced 2D Navier-Stokes equations. We show that by minimizing an appropriate action, we can predict the most probable instanton trajectory between two given non-equilibrium stationary states and estimate the period of its occurrence. This is the first time this approach has been applied to bistability in turbulence.

APPLICATION TO THE 2D STOCHASTIC NAVIER-STOKES EQUATIONS

We consider the 2D stochastic Navier-Stokes equations in a regime of weak forcing and dissipation. In this case, the largest scales of the flow self-organize to produce coherent jets and vortices [3]. Moreover, it was recently shown that over long times, random switchings between two non-equilibrium stationary states can occur – more precisely between a zonal flow and a vortex dipole [2]. It is this behaviour that has given us the motivation to apply instanton theory to the 2D stochastic Navier-Stokes equations. We determine an appropriate action for the system, design an algorithm to compute the instanton trajectories for fluid flows and discuss the difficulties related to the large number of degrees of freedom and the discretization. We will present results for instantons in a variety of scenarios and additional results related to large deviations for the 2D stochastic Navier-Stokes equations. Finally, we will discuss the applicability to more complex turbulent flows that show bistable behavior, such as the Kuroshio ocean current [11].

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