EFFECTS OF MHD TURBULENCE ON MEAN MAGNETIC PRESSURE AND FORMATION OF MAGNETIC STRUCTURES

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Abstract A review of analytical and numerical results on effects of developed magnetohydrodynamic (MHD) turbulence on mean magnetic pressure and formation of magnetic structures is presented. Suppression of turbulent hydromagnetic pressure (the isotropic part of combined Reynolds and Maxwell stresses) by the mean large-scale magnetic field is related to an effective mechanism for the formation of magnetic inhomogeneous structures in MHD turbulence. At large Reynolds numbers and for sub-equipartition mean magnetic fields, the resulting negative turbulent contribution can be enough large so that the effective mean magnetic pressure (the sum of turbulent and non-turbulent contributions) appears negative. We also investigated the effect of mean current density on the turbulent hydromagnetic pressure reduction, and demonstrated that an enhanced mean current density increases the suppression of the turbulent pressure. Such currents are associated with sharp gradients of the growing magnetic structures. The negative effective mean magnetic pressure was found in direct numerical simulation (DNS) in both, stably stratified forced turbulence and turbulent convection. This phenomenon causes the excitation of the negative effective magnetic pressure instability (NEMPI). By the action of this instability, an initially uniform magnetic field forms flux concentrations whose scale is large compared to the turbulent scale. This instability has been recently detected in DNS of forced stratified MHD turbulence that requires enough large scale separation between the forcing scale and the size of the box (e.g., the number of turbulent eddies in the computational domain is about 30). Strong spontaneous formation of large-scale magnetic structures caused by NEMPI, is seen even without performing any spatial averaging. The characteristic time of the instability is comparable to the turbulent diffusion time. We also demonstrated that the magnetic energy of the forming large-scale inhomogeneous magnetic structures is only weakly dependent on the magnetic Reynolds number, provided its value is large enough for the excitation of NEMPI. Our DNS results support mean-field calculations and analytical results which identified this instability. For example, for an isothermal layer the onset of the instability occurs at the same depth that increases with increasing field strength, the growth rate of NEMPI is independent of the field strength, provided the magnetic structures are fully contained within the domain. NEMPI may play a crucial role in the formation of sunspots and active regions in the upper part of convective zones of Sun and stars.

1. Introduction

Turbulence effects generally refer to the occurrence of correlations between components of velocity, temperature, and/or magnetic fields at small scales. A typical example is turbulent viscosity, which results from the spatial exchange of turbulent eddies characterized by velocity correlations. This leads to the dissipation of energy at small scales. However, there is also the possibility of additional non-diffusive turbulence effects known, e.g., in mean-field electrodynamics, namely the α effect (related to the kinetic helicity of turbulent motions), the turbulent diamagnetic or paramagnetic pumping velocities, the Λ effect in rotating anisotropic turbulence (which can lead to the occurrence of differential rotation in cosmic bodies), etc.

Another example of non-diffusive turbulence phenomena is the combined effect of Reynolds and Maxwell turbulent stress tensor that depends on the mean magnetic field, and can lead to a local reduction of the total turbulent pressure and hence to the possibility of self-induced concentrations of large-scale magnetic fields [1, 2]. Such a process may play an important role in the formation of sunspots and active regions in the Sun. The goal of this presentation is to review analytical and numerical results [3]-[8] related to the effect of MHD turbulence on mean magnetic pressure and formation of magnetic inhomogeneous structures.

2. Physics of the effect

The physics of this effect is the following. The kinetic energy density in isotropic turbulence contributes to the total turbulent dynamic pressure twice as much as turbulent magnetic energy density $P_{_T} = \frac{1}{3}\rho\overline{\boldsymbol{u}^2} + \frac{1}{6}\overline{\boldsymbol{b}}^2$, where $P_{_T}$ is the total turbulent dynamic pressure caused by velocity and magnetic fluctuations, $\boldsymbol{u}$ and $\boldsymbol{b}$, respectively; the vacuum permeability is set to be 1, ρ is the fluid density, and overbars indicate ensemble averaging. On the other hand, any rise in local turbulent magnetic energy density must be accompanied by an equal and opposite change of turbulent kinetic energy density in order to obey approximate total turbulent energy conservation, i.e., $\frac{1}{2}\rho\overline{\boldsymbol{u}^2} + \frac{1}{2}\overline{\boldsymbol{b}^2} \equiv E_{\rm tot} \approx$ const. DNS in open systems with boundaries [3] show that when the mean magnetic field $\overline{B}$ is much smaller than the equipartition field strength, $\overline{B}_{\rm eq}$, the total energy is conserved, while when $\overline{B} \leq \overline{B}_{\rm eq}$, the total energy decreases slightly with increasing mean field. This clearly implies that, upon generation of magnetic fluctuations, the total turbulent dynamic pressure shows a reversed (destabilizing) feedback, i.e., $P_{_T} = \frac{2}{3}E_{\rm tot} - \frac{1}{6}\overline{\boldsymbol{b}^2}$, so both an increase of $\overline{\boldsymbol{b}^2}$, as well as an increase of the imposed field, which decreases $E_{\rm tot}$, tend to lower the value of $P_{_T}$. For strongly anisotropic turbulence, this equation for $P_{_T}$ is also valid except for the change of the $1/6$ factor into $1/2$ [2]. This phenomenology was supported by analytical studies using the renormalization approach [1] and the spectral τ relaxation approximation [2], and led to the realization that the effective mean magnetic pressure force (the sum of turbulent and non-turbulent contributions) is reduced and can be reversed for certain mean magnetic field strengths. Under certain conditions (e.g. strong density stratification), this can cause the negative effective magnetic pressure instability (NEMPI) via perturbations of a uniform mean magnetic field in stratified turbulence. NEMPI is a convective type instability that is similar to the interchange instability in plasmas and the magnetic buoyancy instability. However, the free energy in interchange and magnetic buoyancy instabilities is due to the gravitational field, while in NEMPI it is provided by the small-scale turbulence. It was suggested [1, 2] that magnetic flux concentrations in the Sun such as active regions and even sunspots might be formed by this reversed feedback effect.

3. Numerical simulations

The magnetic suppression of the combined Reynolds and Maxwell stresses is quantified in terms of new turbulent mean-field coefficients that relate the components of the sum of Reynolds and Maxwell stresses to the mean magnetic field. These coefficients depend on the mean magnetic field and have now been determined in DNS for a broad range of different cases, including unstratified forced turbulence [3], isothermally stratified forced turbulence [5], and turbulent convection [6]. These simulations have clearly demonstrated that the mean effective magnetic pressure is negative for magnetic field strengths below about half the equipartition field strength. This phenomenon causes excitation of NEMPI, which was first found in mean-field calculations of a stratified layer [1]-[3]. However, those results remained unconvincing until NEMPI was also discovered in DNS of forced stratified turbulence [4], whereby strong spontaneous formation of large-scale magnetic structures was observed even without using any spatial averaging [7, 8]. It was found that NEMPI requires strong stratification: decreasing density stratification scale increases the growth rate of NEMPI. The growth rate of NEMPI increases also with the scale-separation ratio between forcing scale and the size of the box.

During development of NEMPI, an inhomogeneous magnetic structure forms first near the surface, but then the structure propagates downward. When such structures were first seen in the mean-field simulations (MFS) [3], they were originally thought to be artifacts of the model that one would not expect to see in the Sun. However, such structures were later also found in DNS [4, 7, 8], highlighting therefore the strong predictive power of MFS. This effect is consistent with our interpretation that this is caused by negative effective magnetic pressure operating on the scale of many turbulent eddies. Indeed, a local decrease of the effective magnetic pressure must be compensated by an increase in gas pressure, which implies higher density, so the structure becomes heavier and sinks in the nonlinear stage of NEMPI.

The simulations were performed with the PENCIL CODE, [1] which uses sixth-order explicit finite differences in space and a third-order accurate time stepping method. DNS were performed in wide range of fluid and magnetic Reynolds numbers with numerical resolutions up to 512^3, and scale separation from 15 to 30. Most of DNS have been performed for an isothermal stably stratified forced turbulence due to its simplicity, allowing a more thorough investigation of all possible aspects of NEMPI.

Whether or not the particular structures seen in DNS really have a correspondence to phenomena in the Sun, cannot be answered at the moment, because our model is still quite unrealistic in many respects. For example in the Sun, the integral turbulence scale and the turbulent velocity change with depth, which is not currently taken into account in DNS. Also the stratification is not isothermal, but convectively unstable. However, DNS in turbulent convection by [6] have shown that the effective magnetic pressure still has a negative minimum in that case, and it may even be deeper and wider than in the isothermal case. With regards to the production of sunspots, it is likely that NEMPI will shut off before the magnetic energy density has reached values comparable with the internal energy of the gas, as is the case in sunspots. Thus, some other mechanism (like the Parker's magnetic buoyancy instability) is still needed to push the field of flux concentrations into that regime.

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[1] `http://pencil-code.googlecode.com`