MEASUREMENT AND ANALYSIS OF INCREMENTAL AVERAGES OF PASSIVE SCALAR STATISTICS IN GRID TURBULENCE

Laurent Mydlarski[1], Colin Meyer[2] & Luminita Danaila[3]
[1]*Department of Mechanical Engineering, McGill University, Montréal, Canada*
[2]*Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, UK*
[3]*CORIA, Université de Rouen, Rouen, France*

Abstract Although structure functions have been the subject of extensive studies over the years, statistics of averages (i.e., sums, not differences) of turbulent quantities measured over an interval have received very little attention. Furthermore, statistics of incremental averages of passive scalars have never been measured. To this end, we study statistics of incremental averages of passive scalars in active-grid turbulence. The results are presented and discussed, in addition to being compared to those of the hydrodynamic field.

INTRODUCTION

Structure functions are defined as averages of powers of differences of a given turbulent quantity, measured over a separation, r. As an example, the (longitudinal) nth order structure function of velocity (u_α) is defined as follows:

$$\langle(\Delta_r u_\alpha)^n\rangle \equiv \langle(u_\alpha(x+r) - u_\alpha(x))^n\rangle,$$

where u_α is the α component of the velocity fluctuation. Structure functions have been extensively studied over the years given that i) Kolmogorov theory provides explicit predictions regarding their scaling, and ii) they provide information about the small-scale structure of turbulent (hydrodynamic or scalar) fields[3, 4]. However, to further our understanding of turbulence, a similar, but entirely different statistic has been proposed – incremental averages of turbulent quantities, defined as follows:

$$\langle(\Sigma_r u_\alpha)^n\rangle \equiv \langle[\frac{1}{2}(u_\alpha(x+r) + u_\alpha(x))]^n\rangle.$$

Incremental averages have been the subject much less research, especially when compared to structure functions. Of particular interest is the work of Mouri and Hori[1] who found that $\langle(\Sigma_r u_\alpha)^n\rangle$ did not only depend on the large scales of the velocity field, but rather depended on all scales $\geq r$. Furthermore, although some studies of incremental averages of turbulent velocities have been undertaken, incremental averages of scalars (passive or otherwise) have never been studied. This fact motivates the present work, which will present, for the first time, statistics of incremental averages of a passive scalar (temperature in air). We will i) present statistics of incremental averages of a passive scalar in active-grid turbulence over a range of Reynolds numbers, ii) compare these results with those of the velocity field (also measured in active-grid turbulence), and iii) interpret and analyze the differences.

EXPERIMENTAL APPARATUS

The experiments were performed in the low-speed, low-background turbulence, horizontal wind tunnel of the Sibley School of Mechanical and Aerospace Engineering at Cornell University. Its test section is of 91.44 $\times$ 91.44 cm^2 cross section and 9.1 m long. The turbulent velocity field was generated by means of an active grid. The passive scalar field was generated by imposing a mean temperature gradient. The action of the turbulence against this gradient generated the temperature fluctuations, which were small enough to remain passive. The velocity field was measured by hot-wire anemoemtry, and the scalar field by cold-wire thermometry. The details of the experiment can be found in Ref. [2].

RESULTS

We begin by plotting in figure 1(a) the second-order incremental average of temperature ($\langle(\Sigma_r\theta)^2\rangle$, normalized by its variance, $\langle\theta^2\rangle$) as a function of separation, for three Reynolds numbers. As required, all three curves are equal to 1 in the limit of $r \to 0$, and asymptote towards the value of 0.5 as $r \to \infty$. No scaling range is observable due to the nature of this function. (However, one does become observable if $\langle(\Sigma_r\theta)^2\rangle$ is reformulated as $1 - \langle(\Sigma_r\theta)^2\rangle/\langle\theta^2\rangle$. Plotted in this way, the scaling exponent is the same as that for the second-order structure function, $\langle(\Delta_r\theta)^2\rangle$.) We also remark that the oscillation in $\langle(\Sigma_r\theta)^2\rangle$ at large scales for the $R_\lambda = 140$ data set derives from the "synchronous" manner in which the active grid is operated in that case, which results in a low-frequency oscillation observable in the power-spectrum of the scalar. (See Ref. [2] for more details.) In figure 1(b), the second-order incremental averages of u, v and θ are plotted for $R_\lambda = 582$. One observes that the incremental averages of velocity tend towards their large-scale, asymptotic value more rapidly than that of the scalar field (with the transverse velocity being the quickest of the two).

In figure 2, we plot the expectations of incremental averages (of u, v and θ) conditioned upon their respective incremental difference for $r/\eta = 10$. In the limit of $r \to 0$, these can be interpreted as the expectation of the variance of the given

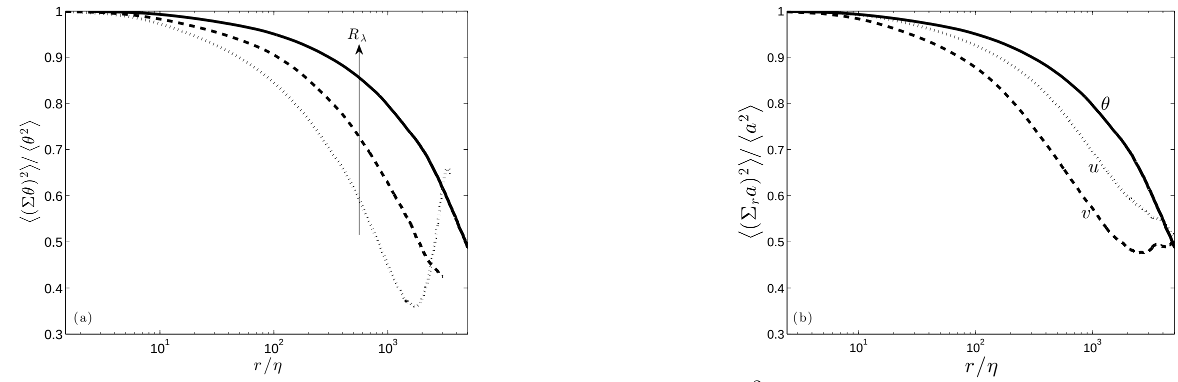

Figure 1. Second-order incremental averages of turbulent quantities. (a) $\langle(\Sigma_r\theta)^2\rangle$, at $R_\lambda = 140$ (dotted line), $R_\lambda = 306$ (dashed line), and $R_\lambda = 582$ (solid line). (b) $\langle(\Sigma_r u)^2\rangle$ (dotted line), $\langle(\Sigma_r v)^2\rangle$ (dashed line), and $\langle(\Sigma_r\theta)^2\rangle$ (solid line) at $R_\lambda = 582$.

quantity conditioned upon its derivative (which is directly related to its dissipation). The behaviours of the conditional expectations of the velocity (figure 2(a)) are significantly different than those of the passive scalar (figure (2b)) – the former being rounded and concave up (similar to those measured in grid turbulence in Ref. [1]), and the latter having a defined, concave-down peak in the middle and being concave up at its edges. Given that incremental averages are dependent on the largest scales of the flow, these observed differences are presumably related to the different nature of the velocity and scalar fields at large scales. Whereas the velocity field is approximately homogeneous and isotropic, with a turbulent kinetic energy budget that balances the decay of turbulent kinetic energy and its rate of dissipation, the large scales of the scalar field are more complex, due to the presence of the mean temperature gradient that results in the production of scalar variance, with a corresponding term in the scalar variance budget.

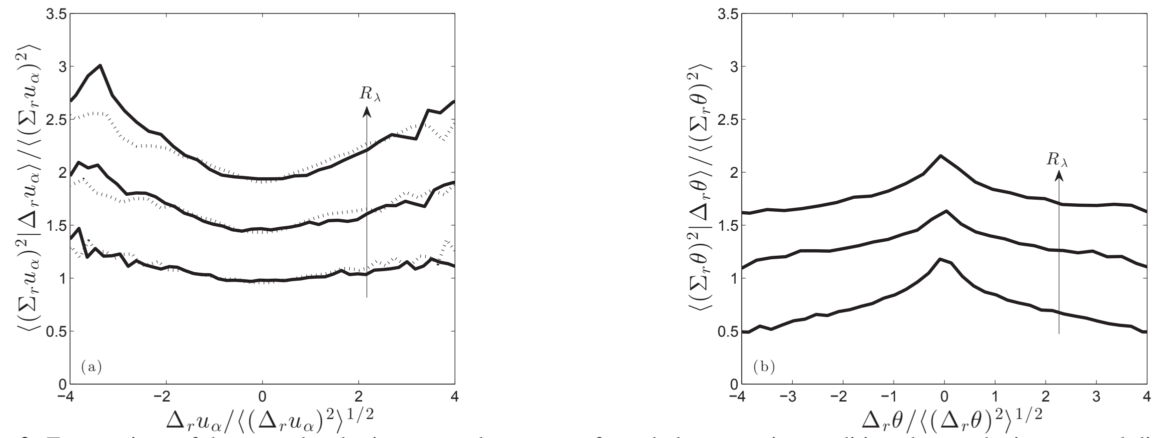

Figure 2. Expectations of the second-order incremental averages of a turbulent quantity conditioned upon the incremental difference for $R_\lambda = 140$, 306 and 582. $r/\eta = 10$. (a) $\langle(\Sigma_r u)^2|\Delta_r u\rangle/\langle(\Sigma_r u)^2\rangle$ (solid line); $\langle(\Sigma_r v)^2|\Delta_r v\rangle/\langle(\Sigma_r v)^2\rangle$ (dotted line). (b) $\langle(\Sigma_r\theta)^2|\Delta_r\theta\rangle/\langle(\Sigma_r\theta)^2\rangle$.

In the full paper, we will present additional results (including higher-order statistics, probability density functions of the incremental averages, additional conditional expectations), and discuss the development of a scale-by-scale budget of the passive scalar averages. We remark that the latter is *not* a trivial exercise in which Yaglom's equation [5] is simply extended to the sum (instead of differences) of a passive scalar. The reason is that the assumption of local homogeneity is no longer valid, as the behaviour of $\Sigma\theta$ is dictated by scales greater than or equal to r, which are, in general, inhomogeneous.

References

[1] H. Mouri and A. Hori. Two-point velocity average of turbulence: Statistics and their implications. *Phys. Fluids*, **12**:115110–1 – 115110–7, 2010.
[2] L. Mydlarski and Z. Warhaft. Passive scalar statistics in high-Péclet-number grid turbulence. *J. Fluid Mech.*, **358**:135–175, 1998.
[3] K. R. Sreenivasan and R. A. Antonia. The phenomenology of small-scale turbulence. *Annu. Rev. Fluid Mech.*, **29**:435–472, 1997.
[4] Z. Warhaft. Passive scalars in turbulent flows. *Annu. Rev. Fluid Mech.*, **32**:203–240, 2000.
[5] A. M. Yaglom. On the local structure of a temperature field in a turbulent flow. *Dokl. Akad. Nauk. SSSR*, **69**:743–746, 1949.