SPECTRAL ANALYSIS OF THE TRANSITION TO TURBULENCE FROM A DIPOLE IN STRATIFIED FLUID

Jean-Marc Chomaz[1], Pierre Augier[1,2] & Paul Billant[1]
[1]*LadHyX, CNRS, Ecole Polytechnique, Palaiseau, France*
[2]*Linné Flow Centre, KTH Mechanics, KTH, Stockholm, Sweden*

Abstract We investigate through numerical simulations the spectral properties of the turbulence generated during the nonlinear evolution of a Lamb-Chaplygin dipole in a stratified fluid for a high Reynolds number $Re = 28000$ and a wide range of horizontal Froude number $F_h \in [0.0225\ 0.135]$ and buoyancy Reynolds number $\mathcal{R} = ReF_h{}^2 \in [14\ 510]$. A spectral analysis shows that this transition is dominated by two kinds of transfers: first, the shear instability induces a direct non-local transfer toward horizontal wavelengths of the order of the buoyancy scale $L_b = U/N$, where U is the characteristic horizontal velocity of the dipole and N the Brunt-Väisälä frequency; second, the destabilization of the Kelvin-Helmholtz billows and the gravitational instability lead to small-scale weakly stratified turbulence. We show that the anisotropic spectra at the maximum of dissipation share many characteristics with those obtained from numerical simulations of forced stratified turbulence and from measurements in the atmosphere and in the ocean. The article presenting this study [2] is the subject of a Focus on Fluids article [10].

THE MECHANISMS OF THE TRANSITION TO TURBULENCE

The evolution of a counter-rotating vortex pair in a stratified fluid has been extensively studied, in particular because it is one of the simplest flow on which the zigzag instability develops and from which the buoyancy length scale naturally emerges as the vertical length [4, 3, 5]. Recently, [6], [11] and [1] have investigated the nonlinear development of the zigzag instability. They have shown that both the shear and gravitational instabilities appear at high buoyancy Reynolds number when the zigzag instability has a finite amplitude leading to a transition to turbulence.

In order to investigate the mechanisms of this transition and to analyse the spectral properties of the turbulence, we have performed a set of high resolution numerical simulations for a high Reynolds number $Re = 28000$. The numerical simulations use a weak hyperviscosity and are therefore almost DNS. Figures 1(*a*,*b*,*c*) present the time evolution of the density field for $F_h = 0.09$ in a horizontal cross-section at the level at which the shear instability appears. By $t = 3.3$, the amplitude of the bending deformations is large but no secondary instability is active yet. At $t = 3.8$, small-scale wiggles can be seen (Figure 1*b*). Eventually, the small-scale turbulence invades a large portion of the domain (figure 1*c*).

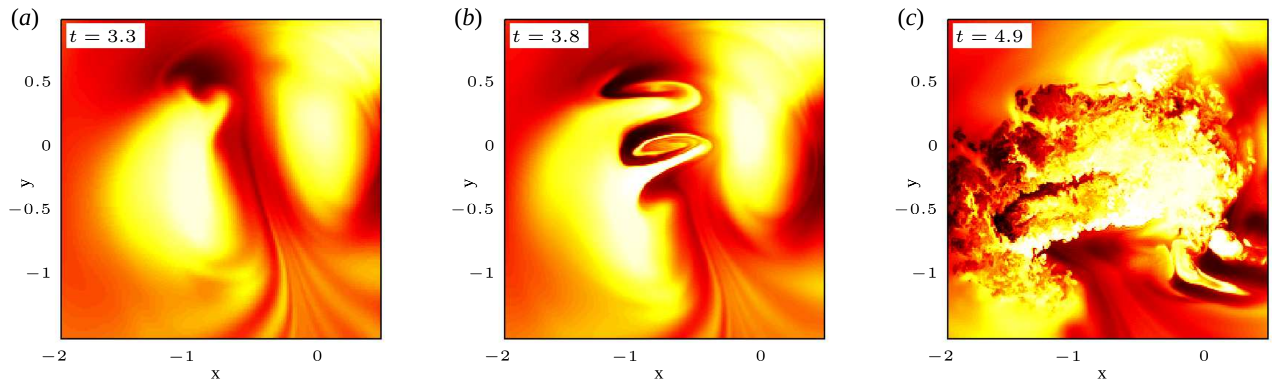

Figure 1. Horizontal cross-sections of the density field at the level at which the shear instability begins to develop for three times.

SPECTRAL ANALYSIS WITH VARIATION OF F_H AND $\mathcal{R}$

Figure 2 presents horizontal (continuous curves) and vertical (dashed curves) compensated kinetic spectra $E_K(k_h)\varepsilon_K{}^{2/3}k_h{}^{5/3}$ and $E_K(k_z)\varepsilon_K{}^{2/3}k_z{}^{5/3}$ obtained from simulations with different values of F_h but the same Reynolds number. The spectra have been time-averaged over $\Delta t = 0.3$ around the time where the total dissipation is maximum. The horizontal spectra of kinetic energy exhibits a $\varepsilon_K{}^{2/3}k_h^{-5/3}$ power law (where k_h is the horizontal wavenumber and ε_K is the dissipation rate of kinetic energy) from $k_b = 2\pi/L_b$ to the dissipative scales, with an energy deficit between the integral scale and k_b and an excess around k_b. The vertical spectra are very steep near $k_z = k_b$ and show a tendency to follow a k_z^{-3} slope. They flatten when approaching the horizontal spectra at large wavenumbers and their slope tends to $k_z^{-5/3}$ except for the highest stratification $F_h = 0.0025$, where the two curves approach each other only in the dissipation range (i.e. the

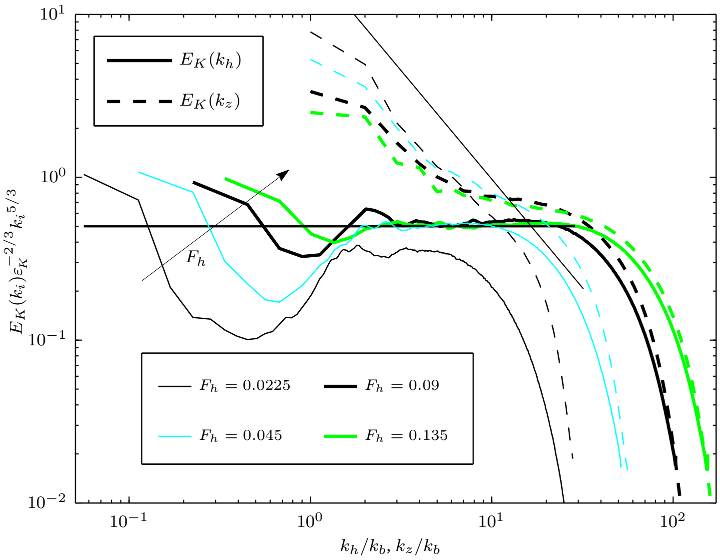

Figure 2. Horizontal and vertical compensated spectra $E_K(k_i)\varepsilon_K^{-2/3}{k_i}^{5/3}$ as a function of the dimensionless wavenumber k_i/k_b for four runs with different values of the Froude number $F_h = 0.0025$, 0.045, 0.09 and 0.135 but the same Reynolds number. Each curve is the average over time interval $\Delta t = 0.3$ near the maximum of the dissipation. The thin straight line indicates the k_z^{-3} power law and the horizontal thick line the $C\varepsilon_K{}^{2/3}k^{-5/3}$ law, with $C = 0.5$.

Ozmidov scale is of the order of the Kolmogorov scale). We have shown that the vertical spectra can be expressed as $E(k_z) = C_N N^2 k_z^{-3} + C\varepsilon_K{}^{2/3}k_z^{-5/3}$ where C_N and C are two constants of order unity.

Thus, the anisotropic spectra share many characteristics with those obtained from numerical simulations of forced stratified turbulence and from measurements in the atmosphere and in the ocean. This is remarkable because the initial flow is very simple and not turbulent. Moreover, the fundamental difference between a transition toward turbulence and developed turbulence has to be stressed. With only two vortices interacting, the dynamics at large horizontal scales is dominated by the zigzag instability and there is no strongly stratified cascade along the horizontal. This contrasts with numerical simulations of forced stratified turbulence which exhibit a forward strongly stratified cascade but for which the overturning motions at the buoyancy length scale and beyond are not resolved or only weakly resolved due to the use of strongly anisotropic numerical meshes [7, 8, 9].

Since the transition in the vertical spectra happens at the Ozmidov length scale, it is tempting to conclude that the overturning motions at the buoyancy scale are strongly anisotropic. However, this is not the case. Indeed, we have shown that the very steep vertical spectrum is mainly due to the large horizontal scales of the dipole that is strongly deformed along the vertical by the zigzag instability. In contrast, the vertical spectrum computed with spectral modes with horizontal wavenumbers larger than the buoyancy wavenumber k_b does not present any k_z^{-3} power law but exhibits a $k_z^{-5/3}$ power law from a vertical wavenumber scaling like the Ozmidov wavenumber k_o down to the dissipative range.

References

[1] P. Augier and P. Billant. Onset of secondary instabilities on the zigzag instability in stratified fluids. *J. Fluid Mech.*, **662**:120–131, 2011.

[2] P. Augier, J.-M. Chomaz, and P. Billant. Spectral analysis of the transition to turbulence from a dipole in stratified fluids. *J. Fluid Mech.*, **713**:86–108, 2012.

[3] P. Billant. Zigzag instability of vortex pairs in stratified and rotating fluids. Part 1. General stability equations. *J. Fluid Mech.*, **660**:354–395, 2010.

[4] P. Billant and J.-M. Chomaz. Experimental evidence for a new instability of a vertical columnar vortex pair in a strongly stratified fluid. *J. Fluid Mech.*, **418**:167–188, 2000.

[5] P. Billant, A. Deloncle, J.-M. Chomaz, and P. Otheguy. Zigzag instability of vortex pairs in stratified and rotating fluids. Part 2. Analytical and numerical analyses. *J. Fluid Mech.*, **660**:396–429, 2010.

[6] A. Deloncle, P. Billant, and J.-M. Chomaz. Nonlinear evolution of the zigzag instability in stratified fluids: a shortcut on the route to dissipation. *J. Fluid Mech.*, **599**:229–238, 2008.

[7] J. N. Koshyk and K. Hamilton. The horizontal kinetic energy spectrum and spectral budget simulated by a high-resolution troposphere-stratosphere-mesosphere GCM. *J. Atmos. Sci.*, **58**(4):329–348, 2001.

[8] E. Lindborg. The energy cascade in a strongly stratified fluid. *J. Fluid Mech.*, **550**:207–242, 2006.

[9] M. L. Waite. Stratified turbulence at the buoyancy scale. *Physics of Fluids*, **23**(6):066602, 2011.

[10] M. L. Waite. The vortex instability pathway in stratified turbulence. *J. Fluid Mech.*, **716**:1–4, 2013.

[11] M. L. Waite and P. K. Smolarkiewicz. Instability and breakdown of a vertical vortex pair in a strongly stratified fluid. *J. Fluid Mech.*, **606**:239–273, 2008.