SCALAR GRADIENT STATISTICS IN ISOTROPIC TURBULENCE IN THE PRESENCE OF A MEAN SCALAR GRADIENT

Wouter J.T. Bos
LMFA, CNRS, Ecole Centrale de Lyon, Université de Lyon, Ecully, France

Abstract We investigate the origin of the non-vanishing scalar gradient skewness in isotropic turbulence on which a mean-scalar gradient is imposed. The problem of the advection of an anisotropic scalar field is reformulated in terms of the advection of an isotropic vector field. It is shown how the scaling of the scalar gradient skewness depends on the choice of the time-scale used for the Lagrangian decorrelation of the vector field. The persistent anisotropy in the small scales for the third order statistics is shown to be perfectly compatible with Corrsin-Obukhov scaling for second order quantities.

INTRODUCTION

In isotropic turbulence in the presence of an imposed mean uniform scalar gradient, the skewness of the scalar gradients seems to tend to a constant, non-zero, value for high Péclet numbers. This phenomenon has puzzled the turbulence community for several decades. Different explanations were given, based on intermittency, ramp-cliff structures, three-point correlators etc. [3]. In the present work we will give a self-consistent description of the phenomenon, relating the gradient skewness to the Lagrangian correlation time of scalar fluctuations. The present approach has the advantage of describing simultaneously the scalar spectra, kinetic energy spectra, scalar flux spectra and scalar gradient skewness, using one single approach.

REFORMULATION OF THE PROBLEM IN TERMS OF AN ADVECTED VECTORFIELD

The dynamics of scalar fluctuations in isotropic turbulence, on which we impose a scalar gradient $\mathbf{\Gamma} = \Gamma \boldsymbol{e}_z$, are governed by the equation

$$\partial_t \theta + u_m \partial_m \theta = D\partial_m^2 \theta + \Gamma u_z. \tag{1}$$

The scalar field is anisotropic due to the anisotropic force term containing the gradient. We reformulate the problem in terms of the vectorfield $\boldsymbol{\chi}$, governed by

$$\partial_t \chi_i + u_m \partial_m \chi_i = D\partial_m^2 \chi_i + \Gamma u_i. \tag{2}$$

The dynamics of equation (1) are recovered by considering only one component of $\boldsymbol{\chi}$. The advantage of this description is that the vectorfield is isotropic, since the source-term is so. To describe the statistics of the advection of this isotropic vector-field we derive closed equations of the EDQNM type, in the spirit of what was done in reference [1] for the velocity-scalar cross-correlation. These expressions are related by kinematic relations [2] to structure functions of the scalar field with separation distances parallel to the direction of the mean-scalar gradient. The resulting equations only necessitate the definition of the Lagrangian correlation time of the vector-field $\boldsymbol{\chi}$.

INFLUENCE OF THE LAGRANGIAN SCALAR CORRELATION TIME

The scalar increment skewness is defined as

$$S_\theta(r) = D_3^{\theta\|}(r) / \left(D_2^{\theta\|}(r)\right)^{3/2}, \tag{3}$$

in which $D_3^{\theta\|}(r)$ and $D_2^{\theta\|}(r)$ are the third and second order scalar structure functions in the direction parallel to the mean gradient.

In Figure 1 we show results for $S_\theta(r)$ for three different Reynolds numbers. In the left figure, we show the results of the numerical integration of our closure in which the Lagrangian correlation time of $\boldsymbol{\chi}$ is defined proportional to the Lagrangian correlation time of the velocity (proportional in the inertial range to $\epsilon^{-1/3}k^{-2/3}$, with ϵ the dissipation rate and k the wave number). The resulting scalar gradient skewness (limit $r \downarrow 0$) tends to zero for increasing Reynolds number (and Péclet number since the Prandtl number is defined unity) unlike experimental results. On the right we have modified the correlation time of the vector field to the integral time-scale. It is observed that the scalar gradient skewness now tends to a value of order unity, as is observed in experiments [3].

From these results it is clear that the scalar correlation time is related to integral statistics and is not given by the Lagrangian correlation time of the velocity field. We note (Fig. 2) that these choices do not change the Kolmogorov-Corrsin-Obukhov scaling for second order statistics.

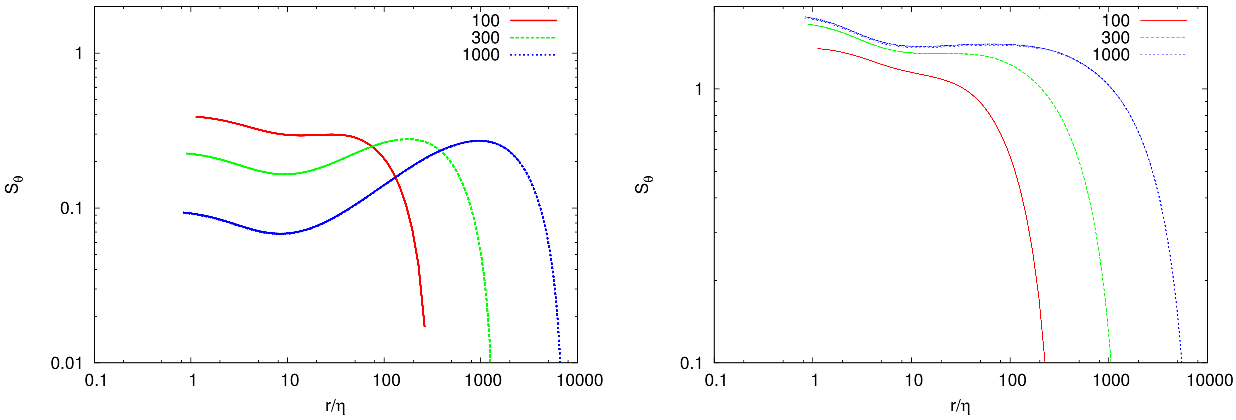

Figure 1. The scalar increment skewness for different Reynolds numbers. Left: when a scale-local correlation time is used in the closure of the vectorfield equation. Right: when the integral time-scale is used to govern the dynamics of the vector field.

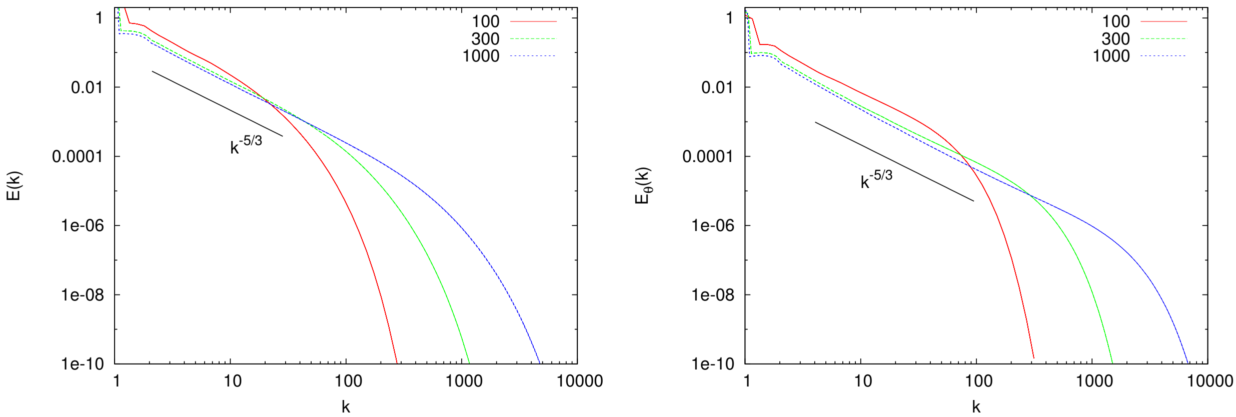

Figure 2. The spectra of the kinetic energy (left) and scalar variance (right) using the (integral) time-scale corresponding to the results in Figure 1 (right).

References

[1] W. Bos and J.-P. Bertoglio. Lagrangian markovianized field approximation for turbulence. *J. Turbul.*, 2013.

[2] W.J.T. Bos, L. Chevillard, J.F. Scott, and R. Rubinstein. Reynolds number effect on the velocity increment skewness in isotropic turbulence. *Phys. Fluids*, **24**:015108, 2012.

[3] Z. Warhaft. Passive scalars in turbulent flows. *Annu. Rev. Fluid Mech.*, **32**:203–240, 2000.