ABSOLUTE INSTABILITIES IN ECCENTRIC TAYLOR–COUETTE–POISEUILLE FLOW

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Abstract Absolute instabilities are investigated in the Taylor–Couette system with axial flow and eccentricity of the cylinder axes, using the Briggs–Bers criterion with a pseudospectral method. Domains of absolute instability are determined for a ratio of radii $\eta = 0.5$, and eccentricities $e \leq 0.7$. Reynolds numbers based on the inner cylinder rotation Re_Ω and mean axial advection Re_z are taken below 50 and 400 respectively. Five modes of instability are studied, respectively the toroidal Taylor vortex, and the left- and right- handed helical vortices of "pseudo-azimuthal" wavenumber 1 and 2, which include the critical modes of the convective instability. A general criterion is presented to discriminate genuine *pinches* from spurious saddle points in the vicinity of third-order saddle points $\partial^2 D/\partial k^2 = 0$ of the dispersion relation D in the complex wavenumber k plane.

PARAMETRIC STUDY OF ABSOLUTE INSTABILITIES

Eccentric Taylor–Couette–Poiseuille flow is a basic model for annular flow of mud involved in the drilling of oil wells. The axial component of the flow is necessary to carry the rock cuttings from the bottom of the well up to the surface, while azimuthal motion is due to drillstring rotation. Eccentricity is caused by the flexibility of the drillstring, slowly shifting its axis along the well. The occurrence of absolute instabilities, namely the propagation of a growing perturbation both upstream and downstream in the well, could cause detrimental transition to complex hydrodynamic régimes, including Taylor vortices, and result in higher pressure drops, increased frictional torque, etc.

Considering axially invariant basic flows and infinitesimal instability waves of complex axial wavenumber k and frequency ω, absolute instabilities are sought in this flow by an analysis of the saddle points $\partial D/\partial k = 0$ of the dispersion relation $D(k, \omega) = 0$ (Huerre & Monkewitz [7, 8]).

In the axisymmetric case, absolute instabilities have already been the object of a number of theoretical and experimental studies, from the initial work of Tsameret & Steinberg [12], Babcock *et al.* [2] to recent analyses including that of Martinand *et al.* [10] and Altmeyer *et al.* [1]. Recent theoretical work concerning the eccentric version of this flow has been concerned with temporal stability [9], which predicts the thresholds for convective instabilities associated to an amplifier behaviour of the flow. The present work complements the description of the linear dynamics by considering the case of hydrodynamic resonance. The same pseudospectral Fourier–Chebyshev method is used in this numerical study involving the incompressible Navier–Stokes equations.

Domains of absolute instability are determined in the four-parameter space involving rotational and axial Reynolds numbers Re_Ω and Re_z, respectively based on inner cylinder rotation and mean axial flow. Geometrical parameters are the ratio of radii η and the eccentricity e. Parametric studies are conducted for $\eta = 0.5$, $e \leq 0.7$, $Re_z \leq 50$ and $Re_\Omega \leq 400$. Only the critical modes responsible for convective instabilities in this range are considered: the toroidal Taylor vortices, and the left-handed helical vortices of "pseudo-azimuthal" wavenumbers $m = 1$ and 2 (defined by continuation of the axisymmetric modes of corresponding azimuthal Fourier component). For the sake of completeness, right-handed helical vortices with $m = 1, 2$ are investigated as well.

CRITERION TO DETERMINE PINCH POINTS IN THE VICINITY OF THIRD-ORDER SADDLE POINTS

Following the Briggs–Bers [3, 4] criterion, absolute instabilities are found by locating the saddle points of the dispersion relation, as mentioned above. For the instability to be causal, it is also necessary that the two *spatial branches* $k(\omega)$ (defined by $D(k(\omega), \omega) = 0$) coalescing at the saddle point be emanating from distinct halves of the complex k plane when ω_i[1] is positive and large enough. When for large ω_i, $k_i > 0$ (resp. $k_i < 0$), the spatial branch is denoted k^+ (resp. k^-) and corresponds to an exponentially damped spatial response to localized forcing for $z > 0$ (resp. $z < 0$). A causal saddle point is termed a *pinch point*, and other saddle points have no influence on the long-term behaviour of the flow. To each saddle point of absolute wavenumber k_0 in the complex k plane corresponds a branch point of absolute frequency ω_0 in the complex ω-plane. The pinch point associated with the maximum absolute growth rate $\omega_{0,i}$ dictates the resonator dynamics of the open flow: an absolute instability will develop if and only if $\omega_{0,i} > 0$.

Dispersion relations of complex systems may contain a large number of saddle points, among which only a few are causal. The subtlety of the Briggs–Bers criterion consists in discriminating the genuine pinch points from other spurious saddle points before finding the most unstable of them. Different pinch points may dictate the dynamics of the flow as parameters

[1] Subscripts "r" and "i" refer to the real and imaginary parts of complex numbers.

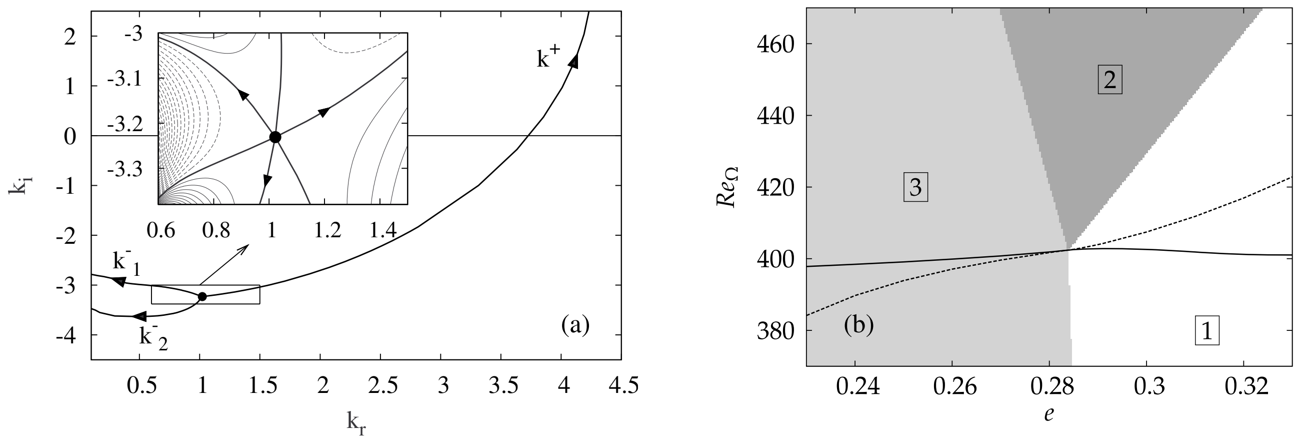

Figure 1. (a) Three k-branches ($k^+/k_1^-/k_2^-$) emanating from third-order saddle point at $k_0 = 1.0223 - 3.2310\mathrm{i}$ for $\eta = 0.5$, $e = 0.285$, $Re_\Omega = 403.21$, $Re_z = 50.115$, and corresponding to contours of constant $\omega_r = \omega_{0,r} = 0.4908$, with ω_i increasing from $\omega_{0,i} = 0$. Solid (resp. dashed) lines in zoom are equispaced contours of constant $\omega_r \geq \omega_{0,r}$ (resp. $\omega_r < \omega_{0,r}$). (b) $\eta = 0.5$, $Re_z = 50$. Solid/dashed lines: zero group velocity threshold for the two saddle points colliding for (k_0, ω_0). Zones: $\boxed{1}$ two pinches k^+/k_1^- & k^+/k_2^-, $\boxed{2}$ one pinch k^+/k_2^- & k_1^-/k_2^-, $\boxed{3}$ one pinch k^+/k_1^- & k_1^-/k_2^-.

are varied. Extensive computations are usually resorted to (e. g. Davies [5], Meliga [11]) in order to uncover the complex dynamics of these flows, as no general theory is available to anticipate the nature of the saddle points.

In eccentric Taylor–Couette–Poiseuille flow, the most absolutely unstable mode is always the propagating Taylor vortex. However, the pinch point dictating the dynamics is found to switch within a set of three distinct saddle points associated to this same mode. For specific parameter values, the saddle points collide by pairs, resulting in third-order saddle points for which $\partial^2 D/\partial k^2 = 0$. Using a Taylor expansion in the neighbourhood of this point, also denoted by subscript "0", it follows that:

$$\omega - \omega_0 \sim \frac{1}{6} \left. \frac{\partial^3 D}{\partial k^3} \right|_0 (k - k_0)^3,$$

showing the coalescence of three distinct spatial branches at the "double saddle point" (see figure 1(a)). For neighbouring sets of control parameters, the "double saddle point" separates into two distinct saddle points, each of them resulting from the collision of two of these three k branches. In order to know if the resulting saddle points are pinches or not, it suffices to determine numerically the nature, k^+ versus k^-, of the spatial branches of the "double saddle point" (or "super branch point", Healey [6]). Using a local development of the dispersion relation and asymptotic matching, the spatial branches forming the simple saddle points can be connected to the three k branches. The method is used to split the parameter space into three distinct regions, depending on the pair of branches colliding (see figure 1(b)).

The method allows significant time savings for predicting absolute instabilities in the presence of multiple saddle points and was applied with success and robustness to the eccentric Taylor–Couette–Poiseuille flow. It was found that unlike convective instabilities, the critical absolutely unstable mode is invariably the propagating Taylor vortex flow, but associated to three different pinch points. Axial flow has a significant stabilising effect for all eccentricities. Surprisingly, at high Re_z, eccentricity is generally destabilising in the sense of absolute instabilities, whereas it is always stabilising in the case of convective instabilities. Critical Re_Ω for absolute instabilities are much higher than for convective instabilities.

References

[1] S. Altmeyer, C. Hoffmann, and M. Lücke. Islands of instability for growth of spiral vortices in the Taylor–Couette system with and without axial through flow. *Phys. Rev. E*, **84**:046308, 2011.

[2] K. L. Babcock, G. Ahlers, and D. S. Cannell. Noise-sustained structure in Taylor–Couette flow with through flow. *Phys. Rev. Lett.*, **67**:3388–3391, 1991.

[3] A. Bers. *Handbook of Plasma Physics*, chapter Space-time evolution of plasma instabilities–absolute and convective, pages 451–517. North-Holland, 1983.

[4] R. J. Briggs. *Electron-stream interactions with plasmas*. MIT Press, 1964.

[5] J. A. Davies. Conditions for absolute instability in the cyclotron resonance maser. *Phys. Fluids B-Plasma*, **1**:663–669, 1989.

[6] J. J. Healey. On the relation between the viscous and inviscid absolute instabilities of the rotating-disk boundary layer. *J. Fluid Mech.*, **511**:179–199, 2004.

[7] P. Huerre and P. A. Monkewitz. Absolute and convective instabilities in free shear layers. *J. Fluid Mech.*, **159**:151–68, 1985.

[8] P. Huerre and P. A. Monkewitz. Local and global instabilities in spatially developing flows. *Annu. Rev. Fluid. Mech.*, **22**:473–537, 1990.

[9] C. Leclercq, B. Pier, and J. F. Scott. Temporal stability of eccentric Taylor–Couette–Poiseuille flow (submitted to *J. Fluid Mech.*). 2013.

[10] D. Martinand, E. Serre, and R. M. Lueptow. Absolute and convective instability of cylindrical Couette flow with axial and radial flows. *Phys. Fluids*, **21**:104102, 2009.

[11] P. Meliga, D. Sipp, and J.-M. Chomaz. Absolute instability in axisymmetric wakes: compressible and density variation effects. *J. Fluid Mech.*, **600**:373–401, 2008.

[12] A. Tsameret and V. Steinberg. Convective vs. absolute instability in Couette–Taylor flow with an axial flow. *Europhys. Lett.*, **14**:331–336, 1991.