LENGTH SCALE TO DETERMINE THE RATE OF ENERGY DISSIPATION IN TURBULENCE

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Abstract The mean rate of energy dissipation $\langle\varepsilon\rangle$ per unit mass of turbulence is often written in the form of $\langle\varepsilon\rangle = C_u \langle u^2\rangle^{3/2}/L_u$, where $\langle u^2\rangle^{1/2}$ is the root-mean-square fluctuation of the longitudinal velocity u and L_u is its correlation length. However, C_u is known to depend on the large-scale configuration of the flow. We define the correlation length L_{u^2} of the local energy u^2 and find that $C_{u^2} = \langle\varepsilon\rangle L_{u^2}/\langle u^2\rangle^{3/2}$ does not depend on the flow configuration. The independence from the flow configuration is also found for the two-point velocity correlation and so on when L_{u^2} is used to normalize the scale.

INTRODUCTION

Since the kinetic energy of turbulence is transferred from large to small scales and is eventually dissipated into heat, the mean rate of energy dissipation per unit mass $\langle\varepsilon\rangle$ is determined by parameters of the large scales [1]:

$$\langle\varepsilon\rangle = C\frac{\langle u^2\rangle^{3/2}}{L}. \tag{1}$$

Here C is a dimensionless coefficient, $\langle u^2\rangle^{1/2}$ is the root-mean-square fluctuation of the longitudinal velocity u along position x, and the length L represents the large scales or equivalently the sizes of the energy-containing eddies. The energy of such eddies is of the order of $\langle u^2\rangle$. Their time scale is of the order of $L/\langle u^2\rangle^{1/2}$. As a result, $\langle u^2\rangle^{3/2}/L$ is of the order of the mean rate at which their energy is transferred to the smaller eddies.

Traditionally, the length L is defined as the velocity correlation length L_u [2], which is obtained by integrating the two-point velocity correlation $\langle u(x+r)u(x)\rangle$:

$$L_u = \frac{\int_0^\infty \langle u(x+r)u(x)\rangle dr}{\langle u^2\rangle}. \tag{2a}$$

However, $C_u = \langle\varepsilon\rangle L_u/\langle u^2\rangle^{3/2}$ is not universal and depends on the large-scale configuration of the flow [3, 4]. That is, C_u is determined by the boundary condition, external force, and so on for the turbulence. The implication is that $\langle u^2\rangle^{3/2}/L_u$ is not proportional to the mean rate at which the kinetic energy is removed from the energy-containing eddies. It is hence concluded that L_u is not proportional to the typical size L of the energy-containing eddies.

This conclusion is important because L_u has been used as a representative of the large scales, not only in studies on Eq. (1) but also in many other studies. It is desirable to find a more universal definition of L. We define L as the correlation length L_{u^2} of the local energy u^2 [5, 6], by integrating its two-point correlation $\langle[u^2(x+r)-\langle u^2\rangle][u^2(x)-\langle u^2\rangle]\rangle$:

$$L_{u^2} = \frac{\int_0^\infty \langle[u^2(x+r)-\langle u^2\rangle][u^2(x)-\langle u^2\rangle]\rangle\, dr}{\langle(u^2-\langle u^2\rangle)^2\rangle}. \tag{2b}$$

For several flows of fully developed turbulence, $C_{u^2} = \langle\varepsilon\rangle L_{u^2}/\langle u^2\rangle^{3/2}$ is studied over a range of the Reynolds number $Re_\lambda = \lambda\langle u^2\rangle^{1/2}/\nu$. Here $\lambda = [\langle u^2\rangle/\langle(\partial_x u)^2\rangle]^{1/2}$ is the Taylor microscale and ν is the kinematic viscosity.

TURBULENCE DATA

The data used here are those of the streamwise velocity fluctuation u obtained experimentally in a wind tunnel [6]. They are for fully developed grid turbulence G1–G5 (Re_λ = 153–436), boundary layer B1–B6 (Re_λ = 455–2097), and jet J1–J6 (Re_λ = 709–3315), among each of which the flow configuration was the same. We obtained these data with a hot-wire anemometer and processed them with Taylor's frozen-eddy hypothesis. Since they are long enough, the resulting statistics are considered to be reliable.

RESULTS

Figure 1 shows $C_u = \langle\varepsilon\rangle L_u/\langle u^2\rangle^{3/2}$ and $C_{u^2} = \langle\varepsilon\rangle L_{u^2}/\langle u^2\rangle^{3/2}$ as a function of Re_λ. Since a log scale is adopted for Re_λ, it is emphasized that C_u and C_{u^2} decrease with an increase in Re_λ. Its values are not high enough for the complete separation of the energy-containing large scales from the energy-dissipating small scales. If Re_λ were increased still more, C_u and C_{u^2} would become independent of Re_λ [3, 4]. More importantly, among the grid turbulence, boundary layer, and jet, while the sequences of C_u do not align [Fig. 1(a)], those of C_{u^2} do align [Fig. 1(b)]. Thus, C_{u^2} is independent of the flow configuration. We favor L_{u^2} as the typical size L of the energy-containing eddies.

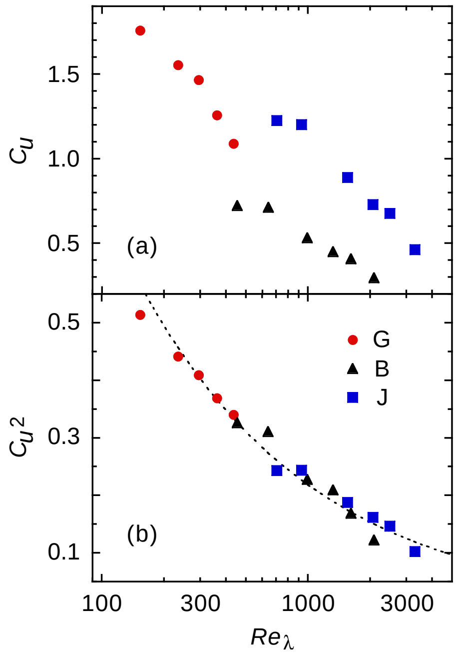

Figure 1. Coefficients C_u and C_{u^2} as a function of Re_λ in grid turbulence G1–G5 (●), boundary layer B1–B6 (▲), and jet J1–J6 (■). The dotted curve is a fit of $C_{u^2} \propto Re_\lambda^{-0.51}$.

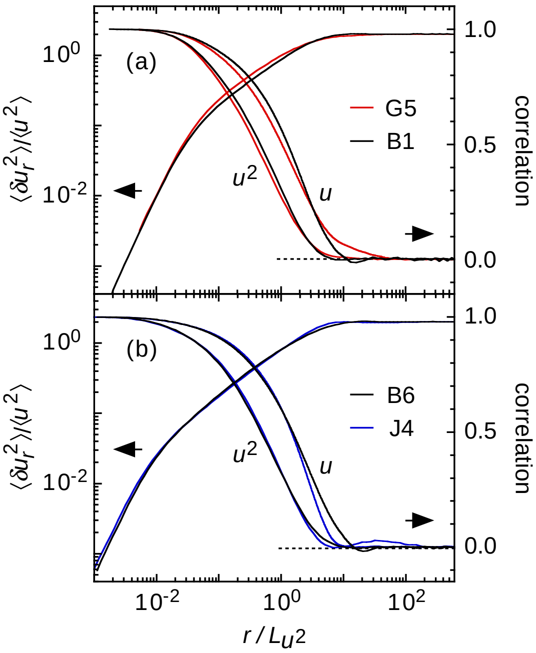

Figure 2. Correlations of u and of u^2 and moment $\langle \delta u_r^2 \rangle$ as a function of r/L_{u^2} in G5 and B1 and in B6 and J4 (solid curves), normalized with the values at $r = 0$.

DISCUSSION

For the mean rate of energy dissipation in the form of $\langle \varepsilon \rangle = C \langle u^2 \rangle^{3/2} / L$, it has been traditional to define L as the velocity correlation length L_u [2]. However, $C_u = \langle \varepsilon \rangle L_u / \langle u^2 \rangle^{3/2}$ depends on the large-scale configuration of the flow [3, 4]. We have defined L as the correlation length L_{u^2} of the local energy u^2, studied $C_{u^2} = \langle \varepsilon \rangle L_{u^2} / \langle u^2 \rangle^{3/2}$ for several flows of fully developed turbulence, and found that C_{u^2} does not depend on the flow configuration [6]. Not L_u but rather L_{u^2} is proportional to the typical size L of the energy-containing eddies, so that $\langle u^2 \rangle^{3/2} / L_{u^2}$ is proportional to the mean rate at which the kinetic energy is removed from those eddies to be eventually dissipated into heat.

The independence from the flow configuration is also found for other statistics when the scale r is normalized with L_{u^2}. Figure 2 shows the two-point correlations and the second-order moment of $\delta u_r = u(x+r) - u(x)$ for pairs of flows where the flow configuration is different but the value of Re_λ is similar. The two curves in each of the pairs are identical at $r \lesssim L_{u^2}$. Thus, L_{u^2} does represent the energy-containing eddies that lie at the top of the energy cascade.

The existing discussions on $C = \langle \varepsilon \rangle L / \langle u^2 \rangle^{3/2}$ often assume that C is independent of the Reynolds number Re_λ [2]. They have to be corrected at $\mathrm{Re}_\lambda \lesssim 10^3$, where we see $C_{u^2} \propto \mathrm{Re}_\lambda^{-\alpha}$ in Fig. 1. The large-scale Reynolds number is $L \langle u^2 \rangle^{1/2} / \nu \propto \mathrm{Re}_\lambda^{2-\alpha}$. The number of degrees of freedom is $(L/\eta)^3 \propto \mathrm{Re}_\lambda^{9/2-3\alpha}$, where $\eta = (\nu^3 / \langle \varepsilon \rangle)^{1/4}$ is the Kolmogorov length. Also if, say, Loitsyansky's invariant holds as $\propto L^5 \langle u^2 \rangle$ in decaying isotropic turbulence, $\partial_t \langle u^2 \rangle \propto -\langle \varepsilon \rangle$ yields the decay law $\langle u^2 \rangle \propto t^{-(10-5\alpha)/(7-5\alpha)}$. These relations could have universal coefficients for $L \propto L_{u^2}$.

Since our data set is limited, our results have to be examined in future with more extensive sets of experimental or numerical data. Nevertheless, it is already certain that L_{u^2} is preferable to the traditional length L_u as the typical size L of the energy-containing eddies or equivalently as the representative of the large scales.

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