# THE THREE-DIMENSIONALITIES OF WALL BOUNDED MHD TURBULENCE

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*Abstract* Flows of electrically conducting fluids in an external magnetic field tend to become 2D. We show experimentally that in channels flows normal to the field and electrically forced through one of the channel walls, the intensity of turbulent fluctuations scales as $Re$ when the flow is quasi-2D, $Re^{2/3}$ when the flow is 3D but strong near both walls, and $Re^{1/2}$ if it is mostly concentrated along one wall only. We also partly confirm the theoretical prediction of [5] that structures of transversal wavelength $k_\perp$ become quasi-2D when they exceed a critical size $k_\perp \sim N_t^{1/2}$, where the true interaction parameter $N_t$ represents the ratio of momentum diffusion by the Lorentz force to inertia for the large scales.

## EXPERIMENTAL SETUP

The principle of our experiment [2] follows that of [4] in which a quasi-2D flow was produced by applying a constant homogeneous magnetic field across a square, shallow container made of electrically insulating walls and filled with liquid metal. The flow was termed "quasi-2D" to reflect its assumed invariance everywhere across the layer (*i.e.* along $B\mathbf{e}_z$), except in *Hartmann* boundary layers that develop along the walls orthogonal to the externally applied magnetic field, called *Hartmann walls*. Unlike this earlier experiment aimed at quasi-2D flows, our container is not shallow, but cubic with inner edge $L(= l_z) = 0.1$ m, so as to obtain 3D flows. The flow entrainment relies on the magnetohydrodynamic (MHD) equivalent of the tornado mechanism: in the same way these are triggered by a vertical flow due to ocean evaporation, columnar vortices of rotation axis $\mathbf{e}_z$ are driven in MHD flows by injecting electric current locally at one Hartmann wall only. 100 current injection electrodes are mounted flush at the bottom Hartmann wall only, arranged in a 10x10 square lattice of step $L_i = 0.1L$, alternately connected to either pole of a DC current generator. Hence, our base quasi-2D flow, obtained for low current and high magnetic field, is a square array of 100 cylindrical, quasi-2D vortices of axis $\mathbf{e}_z$, each of size $L_i \times L$, that rotate in alternate directions. The flow is diagnosed by measuring the electric potential $\phi$ at two sets of 121 points, covering two (3cm)$^2$ squares respectively located on top and bottom Hartmann walls and aligned exactly opposite each other along $\mathbf{e}_z$. Since the electric potential is known not to vary across the very thin Hartmann layers, a quasi-2D flow would yield identical measurements on these two sets, while any difference between the two would betray 3D behaviour. Furthermore, with the electric potential at a wall being proportional to the streamfunction just outside the Hartmann layer [3], estimates for the bulk turbulent fluctuations of velocity near the bottom and top Hartmann walls can be respectively obtained from electric potentials $\phi_b$ and $\phi_t$ measured at bottom and top walls as $U'_b = B\langle\|\nabla\phi_b - \langle\nabla\phi_b\rangle_t\|^2\rangle_t^{1/2}$, and $U'_t = B\langle\|\nabla\phi_t - \langle\nabla\phi_t\rangle_t\|^2\rangle_t^{1/2}$. The flow is controlled by the injected current per electrode $I$ (measured non-dimensionally by a Reynolds number $Re_0 = 2I/(\pi\nu(\sigma\rho\nu)^{1/2})$) and by the externally imposed magnetic field intensity, measured by the *Hartmann number* $Ha = LB(\sigma/(\rho\nu))^{1/2}$. Measurements are performed in established flows of increasingly high $Re_0$ in the interval $[0, 5.1\times10^5]$, for fixed values of Ha in $[1092, 18220]$.

## QUASI-TWO DIMENSIONALITY *VS.* INERTIAL THREE-DIMENSIONALITY DRIVEN BY INERTIA

The variations of Reynolds numbers $Re_b$ and $Re_t$ built on RMS of velocity fluctuations $U'_b$ and $U'_t$ are plotted in figure 1 against $Re_0$. In the low $Re_0$ range, the base vortex array is only just unstable and fluctuations are thus weak. In the limit $Re_0 \to \infty$, by contrast, the flow becomes turbulent and $Re_b$ asymptotically scales as $Re_b \simeq 4\lambda_i Ha^{1/3} Re_0^{2/3}$, where $\lambda_i = L_i/L$. The scaling in $\lambda_i Ha^{1/3}$ reflects an increase in the intensity of velocity fluctuations relative to the mean flow $U_b$ (*i.e* $U'_p/U_b \sim \lambda_i Ha^{1/3}$). For a given value of $Re_0$, turbulent structures indeed become more and more 2D when $Ha$ is increased, and all the more so as they are thick-based (hence the positive exponent of $\lambda_i$). Energy transfer from the main flow to them and through them to smaller scales is then progressively opposed by friction in the Hartmann layer (dissipation of the order of $-\nu U_b^2/l_\perp^2 Ha$), rather than by the stronger Joule dissipation in the bulk of the flow ($\sim -\nu U_b^2/l_\perp^2 Ha^2$). It can be shown that the scaling $Re_0^{2/3}$ corresponds to a phenomenology where electric current is being pulled in the core by finite 3D inertia, instead of travelling vertically from one Hartmann layer to the other, as it does in quasi-2D flows. The Lorentz force associated to this current "leak" balances inertia there and causes three dimensionality. In the absence of this effect, a quasi-2D flow would be expected yield a scaling of the form $Re_b \sim Re_0$.

Fluctuations of velocity near the top wall show a similar behaviour with one important difference. In the limit of High $Ha$, fluctuations follow practically the same law as near the bottom wall, with a small attenuation that reflects three dimensionality. In the presence of three dimensionality, the flow is understandably stronger near the bottom wall, where current is injected than near the opposite one. The most significant difference is visible at low $Ha$ where the flow near the top wall is much weaker than near the bottom wall and obeys a different scaling of the type $Re_b \simeq K(Ha)\lambda_i Ha^{1/3} Re_0^{1/2}$.

At very low $Ha$, the current "leak" from the bottom Hartmann layer has become strong enough for the current in the core to become of the same order as in the Hartmann layer itself. The flow is then entrained over the whole layer in which the current passes and the scaling $Re_b \sim Re_0^{1/2}$ is recovered by equating inertia and the Lorentz force over this layer [1]. The residual flow near the top wall is then entrained by viscous friction through the upper part of the fluid layer where current injected at the bottom wall doesn't penetrate. This layer becomes thicker when $Ha$ increases and this explains that $K(Ha)$ decreases. In this regime, the top wall therefore has very little influence on the bulk.

## SCALE DEPENDENCE OF THREE DIMENSIONALITY

The scale dependence of three dimensionality was evaluated by calculating correlations $C(k_\perp)$ between the time average of spatial Fourier transforms from signals recorded synchronously at the bottom and top walls, along lines located exactly opposite each other. Scaled graphs of these correlations and co-correlations $1-C(k_\perp)$ against wavelength $k_\perp$ (normalised by $L$) are reported on figure 1 (bottom graphs). The scaling with $N_t = Ha^2/(\lambda_i^2 Re_0)$, which represents the ratio of diffusion of momentum along the magnetic field lines by action of the Lorentz force to inertia is only approximate, but all measurements collapse to show that a structure of size $k_\perp$ becomes practically 2D when $k_\perp^c \simeq K_{2D} N_t^{3/5}$. This result confirms, within experimental uncertainty, the theoretical prediction of [5], which would correspond to an exponent of 1/2. Roughly speaking, structures larger than $k_\perp^c$ are 2D while smaller ones are 3D.

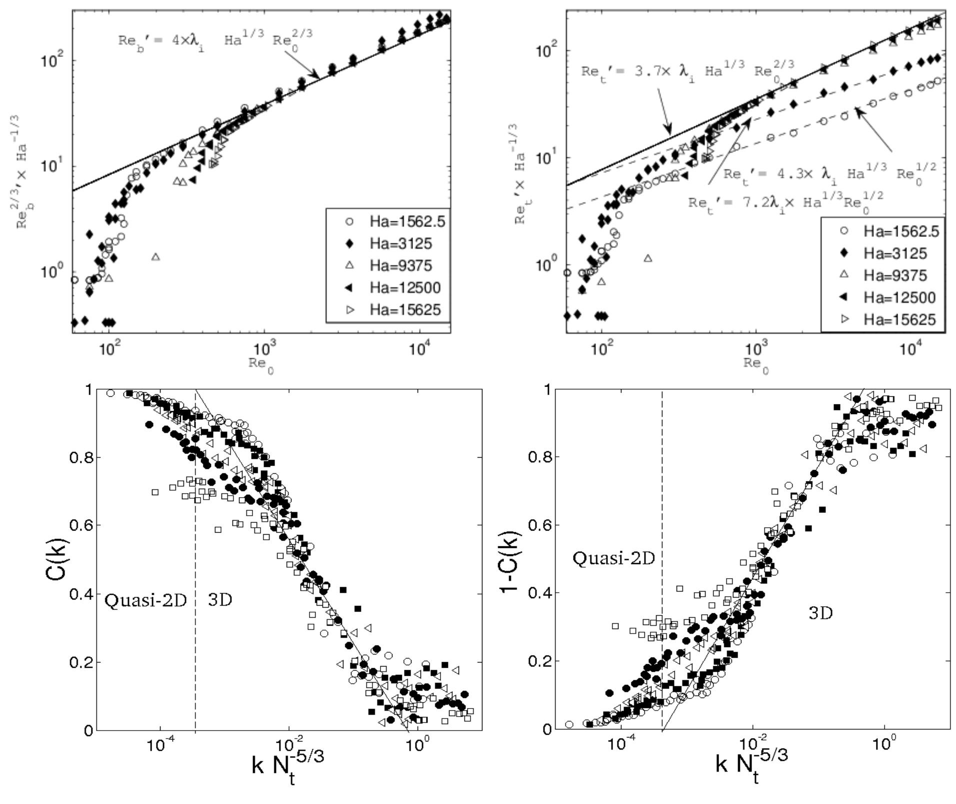


**Figure 1.** Top $Re_b$ and $Re_t$, measuring of the intensity of turbulent fluctuations in the vicinity of the wall where current injected (left) and the opposite walls (right), *vs* a non dimensional measure of the current injected at the bottom wall. Bottom: correlation and co-correlation of Fourier coefficients of $\nabla\phi_b$ and $\nabla\phi_t$.

## References


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