STATISTICAL MECHANICS OF SHALLOW WATER EQUATIONS

Adrien Licari[1] & Max Potters[1] & Antoine Venaille [1] & Freddy Bouchet[1]
[1] *Laboratoire de physique de l'ENS-Lyon, France*

Abstract Statistical mechanics approaches for turbulent geophysical flows is a powerful theoretical tool to predict self-organization of these flows. Previous application of this theory have been restricted to quasi-geostrophic equations. Here we consider the more general shallow water equations that include gravity waves and allow for energy transfer through waves. We show explicit computation of statistical equilibrium states for this model and devise an algorithm to compute these equilibria in more general cases. These results are used to predict the amount of energy that should be transferred into waves or into a large scale geostrophic circulation for a given initial condition.

INTRODUCTION

Geophysical flows are highly turbulent, yet embody large-scale coherent structures, such as ocean rings, jets and largest scale circulation. Understanding how these structures appear and predicting their shape are major theoretical challenges. Statistical mechanics approaches for geophysical flows is a powerful complementary approach to more conventional theoretical or numerical approaches [1]. In the inertial limit, it allows to describe, with only a few thermodynamical parameters, the long time behavior of the largest scales of the flow. Recent studies in quasi-geostrophic models provided encouraging results: a model of the Great Red Spot [2], an explanation of the drift properties of ocean rings [3], the inertial structure of mid basin eastward jets [3], bistability phenomena in complex turbulent flows [4], and so on. Generalization to more comprehensive sets of models, including gravity waves and the possibility of energy transfer through waves motions would be extremely interesting. Those phenomena are indeed essential in understanding geophysical flow energy balance. However, because of essential theoretical difficulties, the previous approaches to describe statistical equilibrium were up to now limited to Quasi-Geostrophic models. The current study fills this gap [7].

RESULTS

The new theory we propose describe geophysical phenomena using statistical mechanics applied to the shallow water model, and easily generalizable to primitive equations. Invariant measures of the Shallow-Water model are built based on the Hamiltonian structure and the Liouville theorem. In parallel with the theory,we devised an algorithm [6, 7] based on the Creutz algorithm [5] (a generalization of Metropolis-Hastings algorithm) in order to sample the microcanonical measures. Numerical simulations are compared to the theoretical results. We apply these new tools in order to describe vortex solutions. Part of the initial energy form large scale coherent structures, whereas the complement is transferred by the gravity waves energy cascade to smaller and smaller scales. We discuss the statistical mechanics prediction of the ratio of energy so transferred to the smallest scales, to the initial energy. This approach is the first theoretical quantitative prediction for this energy transfer.

References

[1] F. Bouchet, A. Venaille *Physics Reports* **515**: 227–295, 2012.
[2] F. Bouchet and J. Sommeria *Journal of Fluid Mechanics* 2002.
[3] A. Venaille, F. Bouchet *Journal of Physical Oceanography*
[4] F. Bouchet and E. Simonnet *Physical Review Letters* , 2009
[5] M. Creutz *Physical Review Letters* **50**: 1411, 1983.
[6] M. Potters, T. Vaillant and F. Bouchet *accepted for publication*, 2013
[7] F. Bouchet, A. Licari, M. Potters and A. Venaille *preprint*, 2013.