Reconnections of quantum vorices

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We demonstrate that a single reconnection of two quantum vortices can lead to creation of a cascade of vortex rings [4, 5]. Our analysis, motivated by the analytical solution in LIA, involves high-resolution Biot-Savart and Gross-Pitaevskii simulations. The latter showed that the rings cascade starts on the atomic scale, with rings diameters orders of magnitude smaller than the characteristic line spacing in the tangle. So created vortex rings may penetrate the tangle and annihilate on the boundaries. This provides an efficient mechanism of the vortex tangle decay in very low temperatures.

We consider quantum vortex lines of constant circulation κ (for the superfluid ^{4}He $\kappa = h/m_{He} = 9.97 \times 10^{-4}\text{cm}^2/\text{s}$). The curve traced out by a vortex filament is specified in the parametric form $\mathbf{s}(\xi, t)$, with t and ξ denoting respectively time and arc length. The vortex local velocity $\dot{\mathbf{s}}(\xi, t)$, given by the Biot-Savart (BS) integral, in some cases can be approximated in terms of the, so called, localized induction approximation (LIA) retaining only the effects of the local vortex curvature;

$$\dot{\mathbf{s}}(\xi, t) = \beta(\mathbf{s}' \times \mathbf{s}'' + \alpha \mathbf{s}''), \tag{1}$$

where the dot and the prime denote the derivatives with respect to t and ξ respectively, β is the self-induction coefficient of order of κ and α is the non-dimensional friction parameter [8]. For $\alpha > 0$, as showed by Lipniacki [6, 7], Eq. 1 has four-dimensional class of self-similar solutions. When the initial vortex configuration consists of two half-lines with a common origin, the line motion is equivalent to a homothety transformation $\mathbf{s}(\xi, t) = \mathbf{S}(l)\sqrt{\beta t}$, with $l := \xi/\sqrt{\beta t}$. In terms of curvature $c(\xi, t) = K(l)/\sqrt{\beta t}$ and torsion $\tau(\xi, t) = T(l)/\sqrt{\beta t}$ the self similar solution can be given in the implicit form.

We reconstructed vortex lines given by that solution using the Frenet-Seret equations to show that for sufficiently small α ($\alpha < 0.44$) and sufficiently small angle between the reconnecting lines $\gamma < \gamma_{LIA}(\alpha)$ the resulting vortex line has two or more self-crossings. Such solutions are not physical, but their existence suggests that the reconnection of two straight vortex lines at a sufficiently small angle may lead to a series of vortex self-reconnections and the creation of a cascade of vortex rings of growing diameter. We confirmed creation of vortex rings cascades by performing high-resolution BS numerical simulations, starting from the configuration which arises shortly after the reconnection of two straight lines. we compare the critical angle $\gamma_{LIA}(\alpha)$ below which analytical solutions have at least two self crossings and the estimated critical angle $\gamma_{BS}(\alpha)$ below which the vortex rings are generated in the BS simulations. Non-local interactions present in the BS simulations enable ring generation for even larger γ than expected from the LIA solutions ($\gamma_{BS}(0) \approx 10.4°$ versus $\gamma_{LIA}(0) \approx 8.5°$). In addition the non-local interactions cause two straight nearly antiparallel vortex lines to approach closely which allow them to undergo reconnection and initiate the vortex loops cascade, Fig. 1A.

Since the BS dynamics may describe the vortex motion before and after the reconnection but not the reconnection event itself, we repeated the simulation of the reconnection of two nearly antiparallel vortices based on the Gross-Pitaevskii (GP) equation [3].

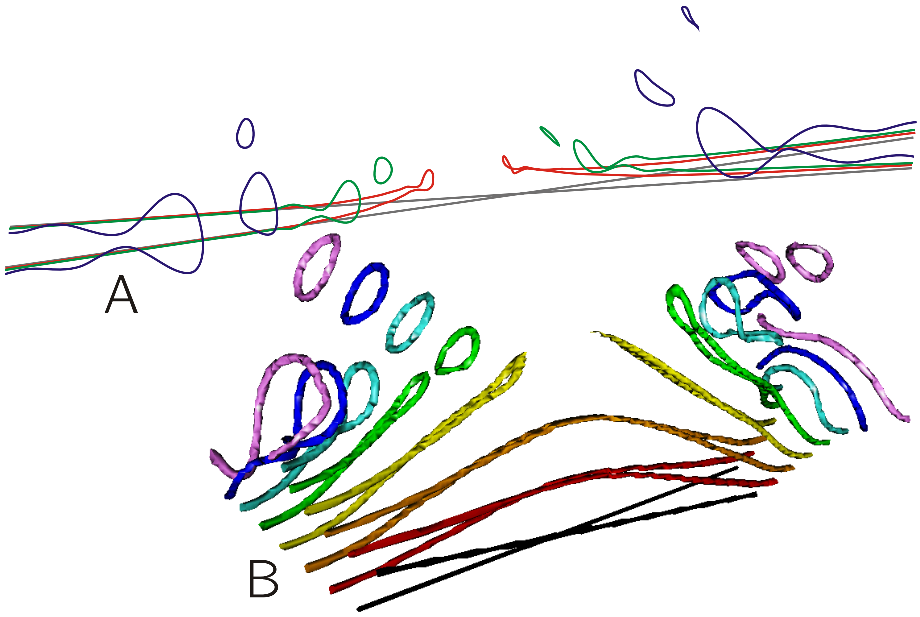

Fig. 1. Biot-Savart A) and Gross-Pitaevskii B) simulations of the reconnection of two initially straight vortex filaments in pure superfluid ($\alpha = 0$), inclined at the angle $\gamma = 5^\circ$. In the BS simulation the initial separation is of $2 \times 10^3 a_s$; the sharp corner arising at the reconnection is smoothed by the arc of radius equal to $20a_s$. In the GP simulation the isosurfaces of condensate density $|\Phi|^2 = 0.3$ are shown. The initial line separation is equal $4a_s$; the two arising rings have radii of the order of $3a_s$.

As shown in Fig. 1B the first ring arising in the cascade has the atomic scale radius of the order of $3a_s$. This explains why the phenomenon was overlooked in the large scale BS simulations of the tangle in which the cusps arising after reconnections are replaced by the arcs of radii comparable to the characteristic radius of curvature of lines in the tangle. Interestingly, in one of the first reconnection studies based on the GP equation, Koplik and Levine [3] showed a transient configuration, possibly preceding the separation of a tiny vortex loop, which either quickly collapsed or left the small simulation box. Recently, emission of small vortex loops due to secondary reconnections was observed by the group of Barenghi [1, 2].

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