ON THE EVOLUTION OF FULL-FIELD STRATIFIED TURBULENCE

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Abstract Decaying turbulence in the presence of a background density stratification is studied by means of direct numerical simulations. The turbulent integral lengthscales are monitored as they evolve in time, and the predictions from recent scaling arguments are tested. Taylor's assumption in the form $\epsilon = u_h^3/l_h$ (where ϵ is the kinetic energy dissipation rate, u_h the horizontal RMS velocity and l_h the horizontal integral lengthscale) is often used in the context of forced stratified turbulence simulations, albeit without confirmation of its validity. Our decaying simulations allow us to test the validity of Taylor's assumption in stratified turbulent flows. Finally, we look at the decay of the horizontal kinetic energy, and find that the decay rate is in line with recent theoretical results.

BACKGROUND

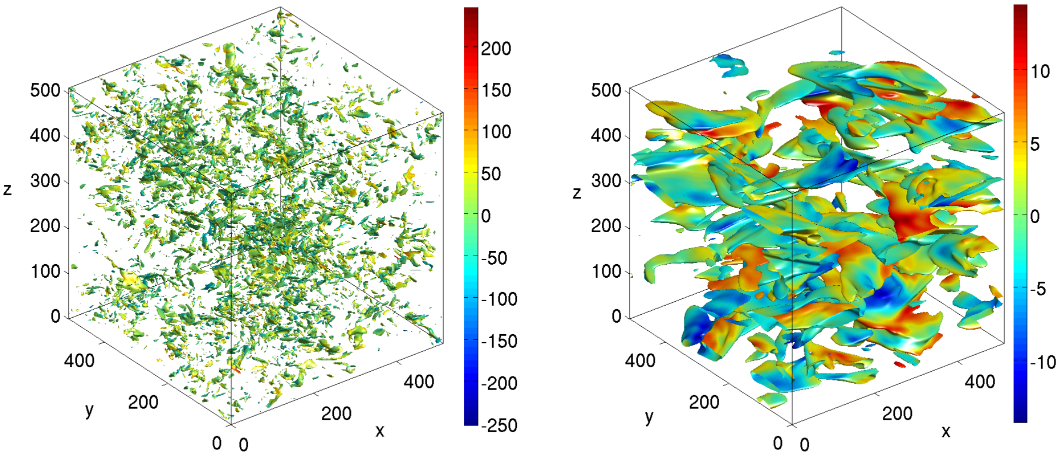
In recent years, a number of scaling arguments have been put forward leading to predictions for the integral lengthscales and energy spectra in stratified turbulence. The first scaling argument leading to predictions on the vertical scale of stratified turbulent flows is given in [1]. The authors start from the Boussinesq momentum and density equations and, using a number of hypotheses, obtain a new set of lowest-order equations. The result $l_z \sim u_h/N$ (where l_z is the vertical scale and N is the Brunt-Väisälä frequency) appears to emerge naturally from their scaling arguments. The analysis is strictly inviscid but the authors point out that the scalings should be valid as long as a certain parameter $\mathcal{R} = \mathrm{Re}_h \mathrm{Fr}_h^2$ is large ($\mathrm{Re}_h = u_h l_h/\nu$ and $\mathrm{Fr}_h = u_h/N l_h$ are the Reynolds and Froude numbers based on horizontal length and velocity scales). Subsequently, another scaling argument was proposed in [4] valid at low Reynolds numbers; in this "viscosity-affected regime" the vertical lengthscale should be given by $l_z \sim l_h/\sqrt{\mathrm{Re}_h}$. Additionally, Taylor's assumption is often used in the stratified turbulence literature, both to derive scaling arguments (as in [5]) or to determine the horizontal lengthscale in forced direct numerical simulations [2]. However, the validity of Taylor's assumption in stratified turbulent flows remains to be confirmed.

There appear to be no systematic studies of decaying full-field stratified turbulence assessing the validity of the above-mentioned scaling arguments, or of Taylor's assumption. Our aim is to cover this gap in the literature. By doing so, we can also test the theoretical predictions of Davidson [3] for the decay of horizontal kinetic energy. In particular, the result of the theory is that the horizontal kinetic energy decreases in time as $u_h^2 \sim t^{-4/5}$ for Saffman turbulence, under the assumption $Fr_z = u_h/N l_z = A$, where A is a constant of order one.

PRELIMINARY RESULTS

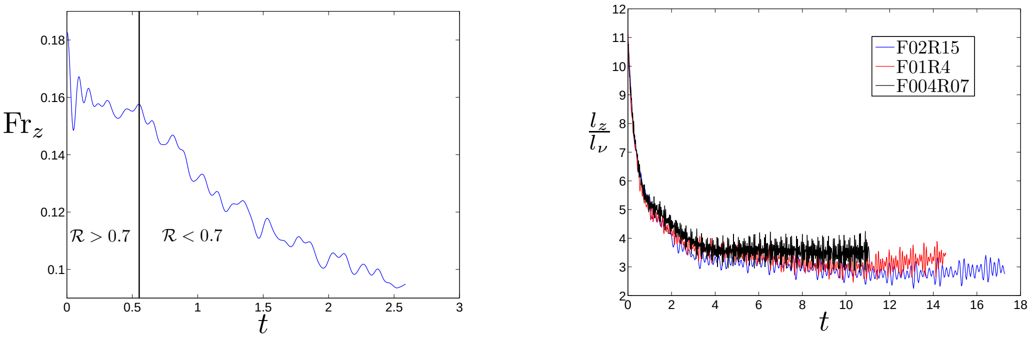
Direct numerical simulations of full-field stratified turbulence have been performed in a periodic cube. The turbulence is initialized using the output of a pre-computation of homogeneous isotropic turbulence, in the absence of stratification, which is run until the velocity derivative skewness S has reached a steady value at $S \approx -0.5$, and the flatness $K > 3.5$. This "fully-developed" field of isotropic turbulence is then subjected to a constant background density gradient, by switching on the Brunt-Väisälä frequency N at $t = 0$. Iso-surfaces of constant vertical vorticity ω_z are shown in Figure 1 for DNS run F0.1R4, corresponding to $\mathrm{Fr}_{h0} = 0.1$ and $\mathcal{R}_0 = 4$ (the 0 subscript on a quantity refers to its value at $t = 0$, when stratification is turned on). The iso-surfaces of ω_z are coloured using the local x-vorticity ω_x; this particular visualisation was chosen to highlight the vertically sheared (hence the ω_x colouring) horizontal motions (containing ω_z) that exist in stratified turbulent flows. At the early time, the ω_z field is composed of worm-like structures typical of isotropic turbulence; the stratification has not had the time, after a single eddy turn-over time, to greatly influence the flow structures. The picture changes dramatically if we inspect the ω_z iso-surface plot at the late time: horizontal pancakes are now present in the flow field, and they are vertically sheared as highlighted by the dark blue and red regions in the iso-surface plot.

The horizontal and vertical integral lengthscales of the turbulence were computed by integrating the velocity correlation functions. The vertical lengthscale was then plotted as a function of time to verify the scaling arguments in [1, 4]. At $t = 0$ we have $\mathcal{R} \gtrsim 1$ for all runs, so according to [2] we should initially be in the strongly stratified turbulence regime, where the scaling $l_z \sim u_h/N$ should be true. Hence it is helpful to look at plots of $\mathrm{Fr}_z = u_h/N l_z$ as a function of time, which should initially exhibit a plateau with $\mathrm{Fr}_z \sim 1$. A plot of the vertical Froude number is given in Figure 2(a) for the case F0.1R4. There appears to be a region close to $t = 0$ where Fr_z is approximately constant, before exhibiting a monotonic decrease with time. This might be considered evidence for a $\mathrm{Fr}_z \sim 1$ region. For most of the duration of our simulations we have $\mathcal{R} < 1$, together with $Fr_h \ll 1$. Therefore the viscosity-affected stratified flow regime should be the physically relevant regime, at least in the later stages of the simulations. To check the viscous scaling in our simulations



(a) (b)

Figure 1. Iso-surfaces of constant ω_z at $\omega_z = |\omega_z|_{max}/5$ coloured by x-vorticity ω_x. Two different times shown, (a) $t/T_0 = 1.0$ (b) $t/T_0 = 24.8$ ($T_0 = l_0/u_0$ is the integral timescale at $t = 0$). The DNS run has a resolution of 512^3 points in the cube.



(a) (b)

Figure 2. Evolution of (a) vertical Froude number Fr_z in run F0.1R4 (b) vertical scale normalized by viscous scale $l_\nu = l_h/\sqrt{\mathrm{Re}_h}$ for cases F0.2R15, F0.1R4 and F0.04R0.7

we plot l_z/l_ν as a function of time and, again, look for a plateau. The results are shown in Figure 2(b); for all cases l_z/l_ν seems to reach a constant value at 3–4. Therefore the prediction is confirmed in our decaying DNS.

Taylor's assumption can be tested by monitoring the evolution of the horizontal integral lengthscale l_h and comparing it to the lengthscale $l_\epsilon = u_h^3/\epsilon$. All runs present a plateau at large times for the quantity l_h/l_ϵ, which would appear to confirm the validity of Taylor's assumption. However it is crucial to test Taylor's assumption at high values of the $\mathcal{R}$-parameter, as this is the regime that is relevant to atmospheric and oceanic flows. Further computations using larger and rectangular boxes are being performed to shed light on this issue.

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