ON THE DISCONTINUOUS TRANSITION TO TURBULENCE IN PLANE COUETTE FLOW

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Abstract Plane Couette flow is a paradigmatic example of wall-bounded shear flow experiencing a direct transition to/from turbulence. The generic features of this transition are discussed using well-resolved numerical simulations in a domain wide enough to undergo laminar-turbulent patterning. Whereas the transition is clearly discontinuous at the global stability threshold $R_{\rm g}$, results at the upper threshold $R_{\rm t}$ – where turbulence becomes featureless – suggest that the transition is also discontinuous, though less dramatically.

Plane Couette flow, the flow developing between two parallel plates moving in opposite directions, is a typical example of wall-bounded shear flow experiencing a direct transition to turbulence. The transition is marked by a coexistence of turbulent and laminar flow in spatially distinct domains. On general grounds this coexistence is observed in some range of Reynolds numbers $[R_{\rm g}, R_{\rm t}]$, called *transitional*. Below $R_{\rm g}$, global stability is achieved and laminar flow is recovered in the long term, whatever the initial state, possibly at the end of a very long turbulent transient. Above $R_{\rm t}$, an essentially featureless turbulent regime is observed.

The definitions of $R_{\rm g}$ and $R_{\rm t}$ have a somewhat conceptual favor and there is still some debate about their exact value in each case. The situation has been clarified recently for pipe flow, at least for $R_{\rm g}$ which was found to correspond to when puff decay is preempted by puff splitting so that turbulence can be sustained [1]. Plane Couette flow has already been much studied, both experimentally [2, 3], numerically [4, 5, 6], and via modeling [7, 8]. The corresponding Reynolds number is defined as $R = Uh/\nu$ where U is the speed of the plates, $2h$ the distance between them, and ν the kinematic viscosity of the fluid. Using this definition, from laboratory experiments, one gets $R_{\rm g} \approx 325$ [2] and $R_{\rm t} \approx 410$ [3]. Along the transitional range, oblique turbulent bands are observed, reminiscent of barber pole turbulence in circular Couette flow [3]. Different values have been proposed for $R_{\rm g}$ and $R_{\rm t}$, illustrating some sensitivity to the methodology.

The time span during which the flow is studied plays an important role and can easily lead to some misestimation of $R_{\rm g}$ since one has to distinguish between sustained and long-lived, with issues similar to what is observed for pipe flow [1]. On the basis of under-resolved simulations, I suggested to understand the decay/growth of turbulence in the neighborhood of $R_{\rm g}$ as the result of the random accumulation of local chaotic fluctuations either breaking bands already formed or controlling the growth of germs [6, c,d], well in line with analogy to the first-order phase transitions put forward by Pomeau [9], and leading to a definition of $R_{\rm g}$ in terms of extreme value theory [10].

The behavior at the putative threshold $R_{\rm t}$ is less clear. A continuous emergence of the pattern seemingly stems from experimental observations for circular Couette flow [3] but the case of plane Couette flow is less well documented, quantitatively speaking. From [4, c], a similar conclusion can be drawn but the bifurcation would take place at a somewhat larger value of $R_{\rm t}$ around 440, which may not easily be reconciled with the observation of intermittent reentrance of the featureless regime observed by the same authors [4, a] (and also in the under-resolved case [6, b]). Based on the data presented below, I suggest that, in contradistinction with the case of circular Couette flow, the transition might be discontinuous. The mentioned reentrance would then be due to the local stability of both featureless turbulence and the banded state, each involving intense fluctuations generated by chaos with large deviations induced by their cumulative effects, like in the neighborhood of $R_{\rm g}$.

Here, I report on numerical simulations performed in the transitional range, more specifically in the vicinity of $R_{\rm t}$, using Gibson's software CHANNELFLOW [11] in a rectangular domain $(L_x, L_z) = (104, 64)$ with periodic boundary conditions. At variance with our previous studies [6], the present computations are appropriately resolved for the considered Reynolds number range, with $N_y = 33$ Chebyshev polynomials in the wall-normal direction, $N_x = 3L_x$ and $N_z = 6L_z$ Fourier modes in the wall-parallel directions (that is $N'_x = 2L_x$ and $N'_z = 4L_z$ de-aliased modes or, in physical space, $\delta x = 0.5$ and $\delta z = 0.25$, comparable to values in [5]).

Various observables can be used to study the transition from a global point of view. An already meaningful information can be obtained from the measure of the distance to laminar flow [6, a], i.e. the root-mean square volume average $\Delta \equiv \tilde{v}_{\rm rms}$ of the perturbation velocity $\tilde{\mathbf{v}} = \mathbf{v} - y\hat{\mathbf{x}}$, where $\hat{\mathbf{x}}$ is the unit vector along the streamwise direction. Visual inspection of the local disturbance energy $\frac{1}{2}\tilde{\mathbf{v}}^2$ averaged over the gap (or a given half-gap) may ease the identification of patterns observed in the transitional range. Assuming that a well-defined pattern develops, the amplitude of its dominant Fourier mode is the most obvious order parameter (see [4, c] and [6, b]). Here Fourier decomposition of the local perturbation energy averaged over the upper half-gap ($0 \le y \le 1$) has been used. Equivalent results are obtained using the lower half-gap ($-1 \le y \le 0$), both slightly better than when considering the flow pattern at a given y (more noisy) or averaging over the full-gap (less noisy but less selective due to the turbulence overhang in the bands [6, b]).

The experiment was started with random initial conditions at $R = 470$ well above the expected value of $R_{\rm t}$. The Reynolds number was subsequently decreased by steps. The initial condition for a given value of R was taken as the state reached by the flow after equilibration at the previous one, i.e., about 500 time units (h/u) except for $R \approx R_{\rm t}$ when the pattern is about to form or formed but fluctuating, which asked for adapted equilibration durations. Animations featuring the dynamics of the perturbation flow field can be downloaded as explained in [12]. Details given in the caption of Fig. 1 strongly suggest that the transition at R_t is discontinuous (curiously as observed in the conceptual Turing model discussed in [8]) while submitted to strong noise of intrinsic origin (local chaos) inducing orientation changes and reentrance of featureless turbulence *via* large deviations. All this prompts to further empirical studies in larger domains (presently underway) and the development of realistic models curing the limitations of our previous attempt [7].

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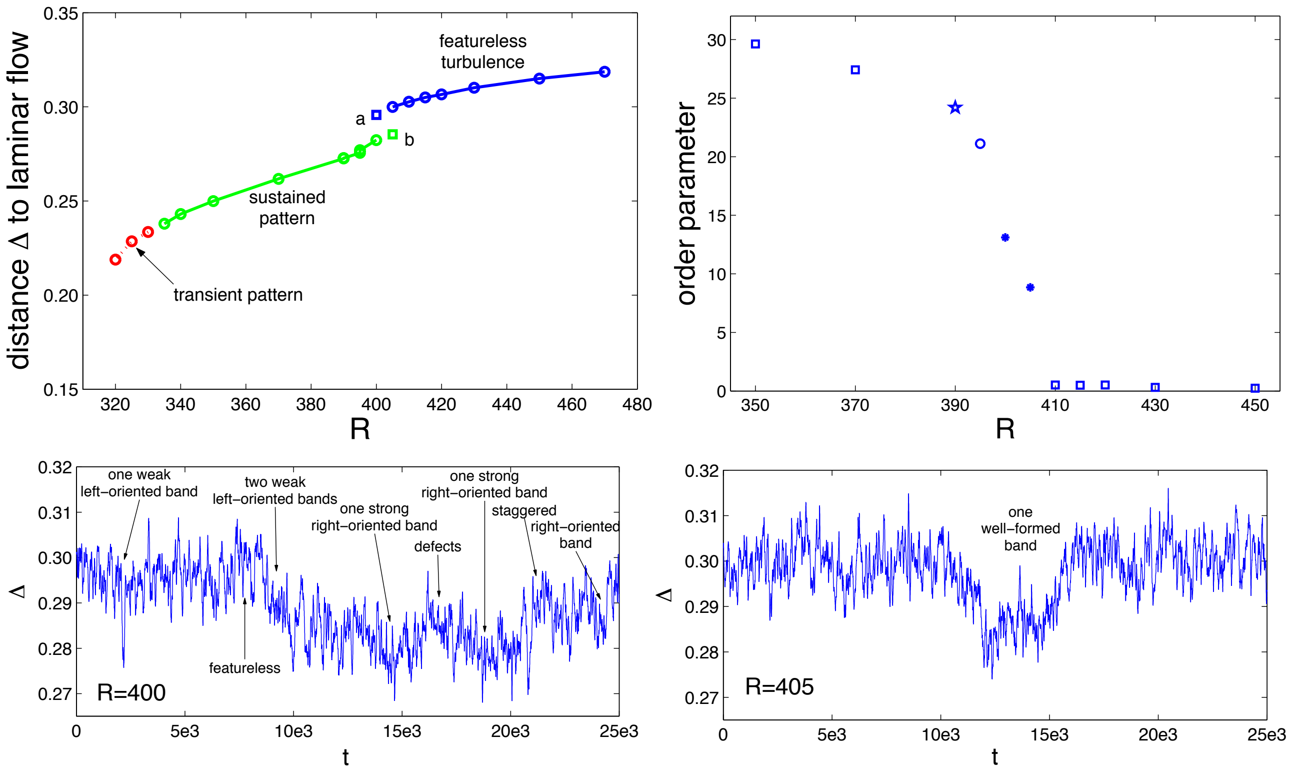

Figure 1. Top-left: Bifurcation diagram displayed in terms of the time-average of Δ as a function of R; values at (a) and (b) correspond to averages extrapolated from specific intermittent configurations: mostly featureless at the beginning of time series for $R = 400$ (bottom-left) and well-formed band around $t = 12500$ for $R = 405$ (bottom-right), respectively.
Top-right: Order parameter (arbitrary units) as function of R; at $R = 390$ (star) an orientation change was observed without significant variation of the order parameter; open circle at $R = 395$ corresponds to a (meta)stable band exceptionally obtained by increasing R from $R = 390$; asterisks at $R = 400$ and 405 give the value of the order parameter, conditioned by the fact that a single band is present.
Bottom: Distance Δ to laminar flow as a function of time (in units of h/u) for $R = 400$ (left) and $R = 405$ (right).