A MIXED MULTISCALE DYNAMIC SGS MODEL ACCOUNTING FOR THE CROSS-TERM

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Abstract Purely dissipative eddy-viscosity subgrid-scales (SGS) stress models are broadly used in large eddy simulations (LES) of turbulent flows. The actual SGS behavior is however locally not purely dissipative. In this sense, more advanced models are desirable. Such a model is presented hereafter and is tested in decaying homogeneous isotropic turbulence (HIT). We propose a dynamic mixed model that accounts for both the Reynolds term and the cross-term of the SGS stress tensor. The model is shown to behave in good agreement with the reference DNS.

GOALS AND MOTIVATIONS

We consider large-eddy simulation (LES) of turbulent flows. Most popular approaches for the subgrid-scale (SGS) stress modeling are, in one way or another, based on a purely dissipative formulation. Typically, the SGS stress $\widetilde{T}_{ij} = \widetilde{u_i u_j} - \widetilde{u}_i\widetilde{u}_j$ is modeled as $\widetilde{T}_{ij}^M = -2\nu_e \widetilde{S}_{ij}$, with $\widetilde{S}_{ij}$, the resolved strain rate and with a closure equation for the SGS eddy viscosity ν_e. The simplest of those is the model by Smagorinsky [6], using $\nu_e = C\Delta^2|\widetilde{S}|$ with Δ, the local effective grid size. A dynamic version was later proposed by Germano et al. [2]. Attempts at discriminating between the scales of the flows were made by Hughes et al. with the so-called variational multiscale (VMS) model [3]. It was designed so as to focus the dissipation on the smallest scales of the LES field only, and was performed using a sharp Fourier cut-off to discriminate between large and small scales of the LES field. Regularized versions thereof were later developed; also the "regularized variational multiscale" (RVMS) model by Jeanmart and Winckelmans [4]. These models have the form $\widetilde{T}_{ij}^M = -2\nu_e \widetilde{S}_{ij}^s$, where the small scales $\widetilde{u}_i^s$ are obtained from an appropriate high-pass filter developed so as to be easily and efficiently applied in physical space. The effective viscosity ν_e is itself evaluated using either $|\widetilde{S}|$ or $|\widetilde{S}^s|$ and the local effective grid size. Those models are still solely dissipative and amount to a more complex diffusion operator.

We recall that the SGS stress tensor is actually made of two distinct parts: the Reynolds term and the cross-term: $\widetilde{T}_{ij} = \widetilde{u_i' u_j'} + (\widetilde{\widetilde{u}_i u_j'} + \widetilde{u_i' \widetilde{u}_j}) = \widetilde{R}_{ij} + \widetilde{C}_{ij}$. The spectral behavior of those two terms is intrinsically different, as also shown by Wang and Oberai [7]. See also Fig. 1 in the present study. This motivates developing a mixed model including a separate closure for each term. The Reynolds term mainly behaves as a purely dissipative operator, locally and globally, whereas the cross-term has a more complex behavior: it is globally dissipative (also at all wavenumbers, see Fig 1), yet it can have significant local backscatter. It is thus reasonable to use one of the aforementioned purely dissipative models for the Reynolds term and to focus our additional effort on modeling the cross-term.

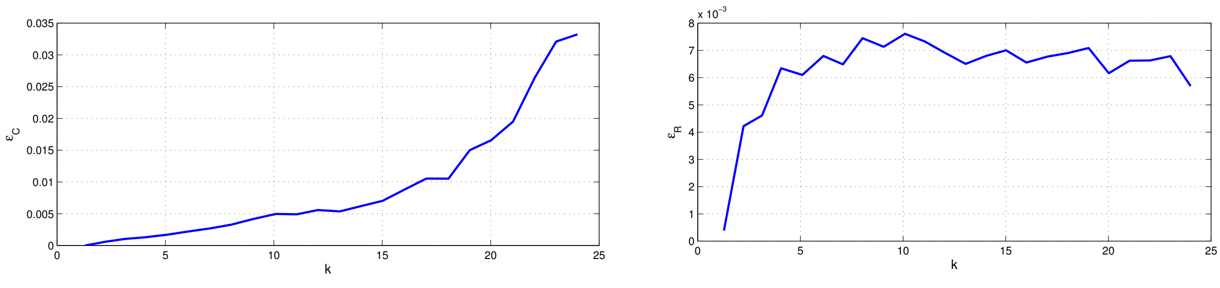
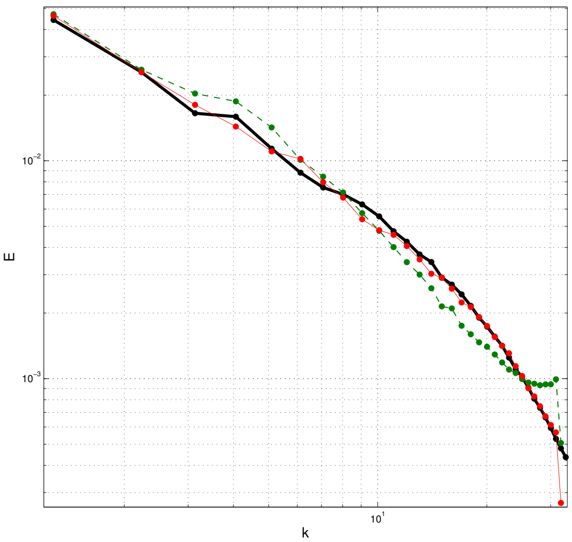


Figure 1. A priori analysis of the dissipation spectrum due to the cross-term (ε_C) and to the Reynolds term (ε_R) for a 48^3 field truncated from a 512^3 DNS field of HIT at $Re_\lambda = 150$.

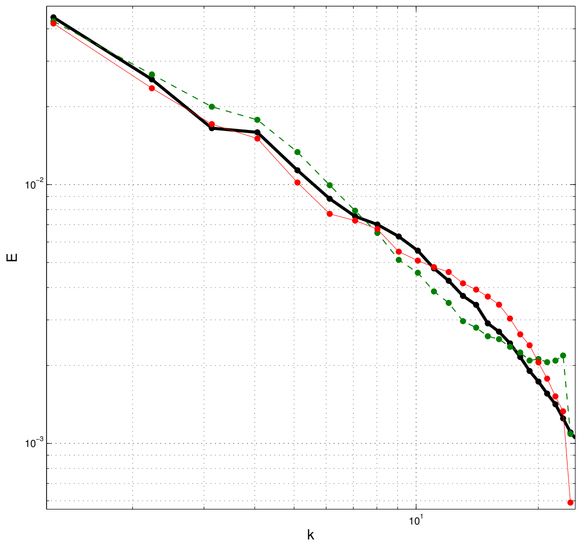
A DYNAMIC MIXED MODEL

The final model has the general form $\widetilde{T}_{ij}^M = \widetilde{R}_{ij}^M + \widetilde{C}_{ij}^M = C_r \widetilde{r}_{ij}^M + C_c \widetilde{c}_{ij}^M$. As previously stated, we use either a Smagorinsky or a RVMS model for the Reynolds term. The cross-term model is based on a scale-similarity argument, with a further modification in order to obtain Galilean invariance. This model is also somewhat linked to the scale-similarity model of Bardina [1]. Several discrete filters (sine filters, implicit filters, sharp truncations) and variants of the model are investigated in a priori analyses. We also keep in mind that one constraint is that the model must be efficiently usable in physical space. The C_c coefficient is calibrated in such a way that the average dissipation of the cross-term model is initially the same as that of the cross-term from the reference DNS. The cross-term model is seen to be quite good, apart from a small lack of dissipation at the smallest scales of the LES grid. This flaw can, in part, be corrected by preferring the RVMS model to the Smagorinsky model in the choice for the Reynolds term model. The Reynolds term model constant remains to be determined. Two approaches are compared: constant C_r (i.e. "calibrated") and dynamic C_r.

A dynamic method is thus also developed in order to determine C_r on the fly. It is here done in a way so as to obtain the same average dissipation as that of a dynamic Smagorinsky model used solely and computed on the same LES field. This methodology is the same as that proposed by Park and Mahesh [5]. LES of decaying HIT are performed with various levels of truncation, filters and Reynolds term models. The results demonstrate the good performance of the dynamic mixed model: examples are provided in Fig. 2. For both 64^3 and 48^3 (i.e. more challenging) LES, the obtained spectrum is closer to that of the reference DNS than when using the dynamic Smagorinsky model, over the range of small to medium wavenumbers. For the 64^3 LES, it is actually close to the reference DNS over the whole range of wavenumbers.



(a) 64^3



(b) 48^3

Figure 2. Energy spectra of LES of decaying HIT at time $tU_{\mathrm{rms},0}/\mathcal{L}_0 = 3.6$. The initial field was that of a truncated 512^3 DNS at $Re_\lambda = 150$: DNS (bullets connected by thick solid), dynamic mixed model (bullets connected by thin solid) and dynamic Smagorinsky model (bullets connected by dash).

References

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