SYMMETRY OF VORTICES IN TRANSITION OF PLANE COUETTE FLOW AT MODERATE REYNOLDS NUMBER

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Abstract The shooting method, which was established originally as a tool to find out an unstable steady solution of subcritical shear flows (cf. J. Phys. Soc. Jpn. 70, 703 [1]), is applied to outline a structural aspect of laminar-turbulent transition of minimal plane Couette flow at the moderate Reynolds number. By adopt as initial conditions the points on the plane in phase space spanned by three distinct exact steady solutions, including the Hairpin Vortex Solution (HVS, cf. Phys.Rev.Lett.102,114501 (2009)), a plenty of trial calculations based on the method are carried out. The result implies that HVS is on the boundary separating the laminar and the turbulent attractors of the plane Couette flow, and that one of the unstable manifolds of HVS constitutes the boundary. Moreover, the asymptotic behaviour of HVS at higher Reynolds number is investigated, which enables us to quantify the role of HVS in the laminar-turbulent transition of Plane Couette flow at the moderate Reynolds number.

INTRODUCTION

Hairpin vortex has been known as one of predominant vortex structures in turbulent shear flows. A corresponding exact solution in plane Couette flow, Hairpin Vortex Solution (HVS)[2], was solved at $Re = 200$ recently by the authors. Specially, the upper branch of the solution contains vortex structures with the shape of a hairpin observed ubiquitously in turbulent shear flows, and satisfies the reflection symmetry towards the spanwise direction. At the leg of the vortex structures, the localised vorticity lifts up the low-speed momentum fluid near the boundary so as to form a couple of low-speed streaky regions apart from the boundary and, at the same time, the head of those induces coalescence of these low-speed streak structures, which can then be visualised as streaky bulges beneath the head of the structure. While the upper branch of HVS has recently attracted researcher's interest in this field due to the characterisation of its spatial structure [3], the lower branch of HVS has been left out of the analysis of the HVS solutions.

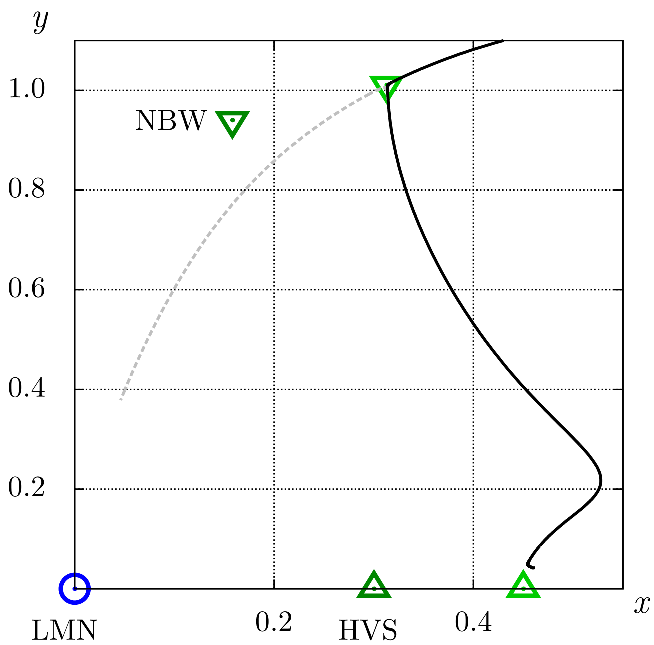

Figure 1. A trajectory of plane Couette flow obtained by the shooting method is projected on the plane through the three exact solutions, the laminar(LMN), NBW, and HVS in phase space at $Re = 10^3$. The boundary separating the laminar and the turbulent attractors, basin boundary, is emerged out of a tangle of trajectories in phase space, as if a ridge line in mountainous area is outlined by a lot of rivulets on the map.

METHOD AND RESULT

In the present study, our focus is on the lower branch of HVS and on its manifold of plane Couette flow at higher Reynolds number ($Re \sim 10^4$). The shooting method, which was established originally as a tool to find out an unstable steady solution of subcritical shear flows[1], is applied to outline the manifold of the lower branch of HVS. It shows us an aspect of laminar-turbulent transition of plane Couette flow at the moderate Reynolds number. Here, the initial conditions used for trials of the method are taken from the points on the plane through the three distinct exact steady solutions of plane Couette flow, including the lower branch of HVS, in phase space. The boundary separating the laminar and the turbulent attractors, which is hereafter named as basin boundary, is emerged out of a tangle of trajectories in phase space, as if a ridge is outlined by a lot of small streams in mountainous area (Fig. 1).

CONCLUDING REMARKS

From the result, firstly, it is concluded that the lower branch of HVS is on the basin boundary at a higher Reynolds number. Secondly, it is inferred that, while one of unstable manifolds of HVS connects to the laminar state, another manifold of HVS connects to another exact solution of plane Couette flow, which was previously obtained by Nagata, Busse & Clever, Waleffe (we used the acronym "NBW" here), the so-called Nagata's solution of plane Couette flow. This implies that a hetero-clinic orbit of these solutions constitutes the basin boundary separating the laminar and the turbulent attractors in phase space of plane Couette flow. Moreover, the asymptotic behaviour of HVS at higher Reynolds number will be investigated, which enables us to quantify the importance of HVS in the laminar-turbulent transition of Plane Couette flow at the moderate Reynolds number.

ACKNOWLEDGMENTS

We would like to dedicate the paper to our families, whose continued support enabled us to research in the field of canonical shear flows. In particular we are grateful to our fathers who over the years installed on us the enthusiasm to pursue vigorously the unknown in order to establish the truth. T.I. is grateful for the financial support received from Aston University under the Visiting Scholars Fund Scheme and EPSRC (GR/S70593/01), which allowed him to spend time in the United Kingdom, while working on this project. T.A. was supported by the European Union FP7 people Marie Curie International Incoming Fellowship grant: T2T-VDG F7-PEOPLE-2011-IIF 298891 . This work has been also supported in partly by KAKENHI (23760164) and partly by the Kansai University Special Research Fund, 2010-2011.

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